Fundamentals of Post-Tensioned Concrete Design for Buildings

Part Three

by

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Overview of This Course

This is Part Two of a two-part course that covers the fundamentals of post-tensioned concrete design for building structures using unbonded tendons. This course is intended as an introductory course for structural engineers new to post-tensioned concrete design and is a refresher for experienced structural engineers. It is assumed that Part One of this course has been successfully taken by the user of this Part Two. By successfully completing this two-part course, you should be comfortable performing a preliminary design by hand and be able to quickly check a computer generated design or an existing design by hand or by simple analysis techniques.

Part One gave a brief historical background and post-tensioned members were differentiated from pre-tensioned members. You learned about the load balancing concept, hyperstatic moments, pre-stress losses, the basic requirements of ACI 318-08 (Building Code Requirements for Structural Concrete), and nominal flexure and shear capacities of post-tensioned members.

In Part Two, we worked several examples of two of the structural systems commonly used in buildings and parking structures, namely a one-way continuous slab, a continuous beam, and a two-way slab. Part Two of the course was example-intensive, and key concepts were introduced along the way.

Part Three we will now continue with the study of two-way, post-tensioned slab systems, including a complete design example using the Equivalent Frame concept. Part Three also covers related topics such as punching shear for two-way slabs and moment transfer at the column. Part Three is an example-intensive course, with key concepts introduced along the way.

The user of this course material must recognize that pre-stressed concrete design is a very broad topic and that only certain fundamentals in a specific area are covered here in this course. It is not intended, nor is it possible within the confines of this course, to cover all aspects of pre-stressed concrete design. It is not intended that the material included in this course be used for design of facilities by an engineer who is inexperienced in pre-stressed concrete design without oversight and guidance from someone more experienced in this field. The author of this course has no control or review authority over the subsequent use of this course material, and thus the author accepts no liability for damages that may result from its use.
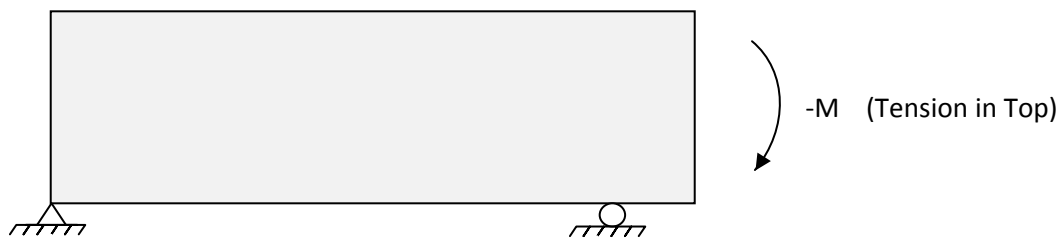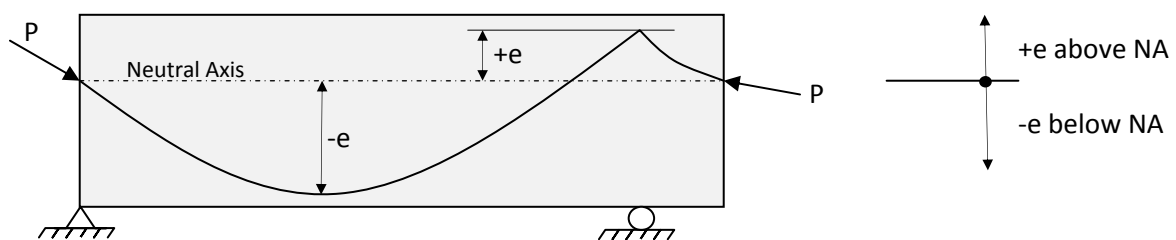
A Word About Sign Conventions

In this course, moments causing tension in the bottom fiber are considered positive. Moments causing tension in the top fiber are negative. Eccentricities below the neutral axis are negative and above the neutral axis they are positive. These sign conventions are illustrated below.

+M   (Tension in Bottom)

Positive Moment

-M   (Tension in Top)

Negative Moment

P

Neutral Axis

+e

-e

P

+e above NA

-e below NA

Eccentricity

Two-Way Slab Systems

Two-way post-tensioned slab systems have been widely used in building construction for many decades. The advantages are many, when compared to non post-tensioned systems, and include thinner slabs, lighter structures, lower floor-to-floor heights, less mild reinforcing steel, and less short and long term deflections.

Two-way flat *slabs* have drop panels, or capitals, at the columns for greater punching shear resistance, while two-way flat *plates* have no drop panels. Contractors generally prefer flat plate construction since the formwork is simple. The thickness of flat plates is often governed by punching shear.

ACI 318 requires moments and shears in two-way post-tensioned slab systems to be determined either by the equivalent frame method (EFM) or by "more detailed design procedures," usually taken to mean finite element methods (FEM). Note that the Direct Design Method may not be used for two-way, post-tensioned slab systems. Because this course is intended to illustrate simplified analysis methods, we will use the EFM exclusively.

<u>Banded Tendons</u>

When designing two-way, post-tensioned slab systems, in addition to the Code provisions and various checks, some of which we have seen in the previous one-way examples in parts one and two of this course, there is an important consideration that simplifies the construction process. Imagine a multi-span two-way slab system with tendons draped in two perpendicular directions. Installing tendons in this configuration would be very difficult, and would be like weaving a basket. Tests have shown that all of the tendons in one direction may be placed and draped in a narrow band on the column centerline in one direction, effectively forming a "beam" within the slab, and the tendons in the perpendicular direction can be uniformly spaced and draped over the banded tendons. Thus, a banded system behaves like two-way system, and it is practical to install the tendons in the field. Essentially all two-way post-tensioned floor systems in use today utilize banded tendons in one direction.

Below is a partial floor framing plan, such as for a hotel. A two-way, banded, flat plate system is shown. The thickness of a two-way, post-tensioned slab for typical hotel column patterns, similar to that shown, is usually around six to eight inches. In the figure below, the banded tendons have been chosen to run across the short direction of the floor plate, forming a three-span condition. The tendons in the perpendicular

direction are uniformly distributed across the width of the building. Depending on the length of the hotel floor plate, these uniformly distributed tendons may need to be stressed at an intermediate construction joint. Practical considerations, such as the maximum length between construction joints, tendon layout, recommended location of pour strips, etc. are beyond the scope of this course.
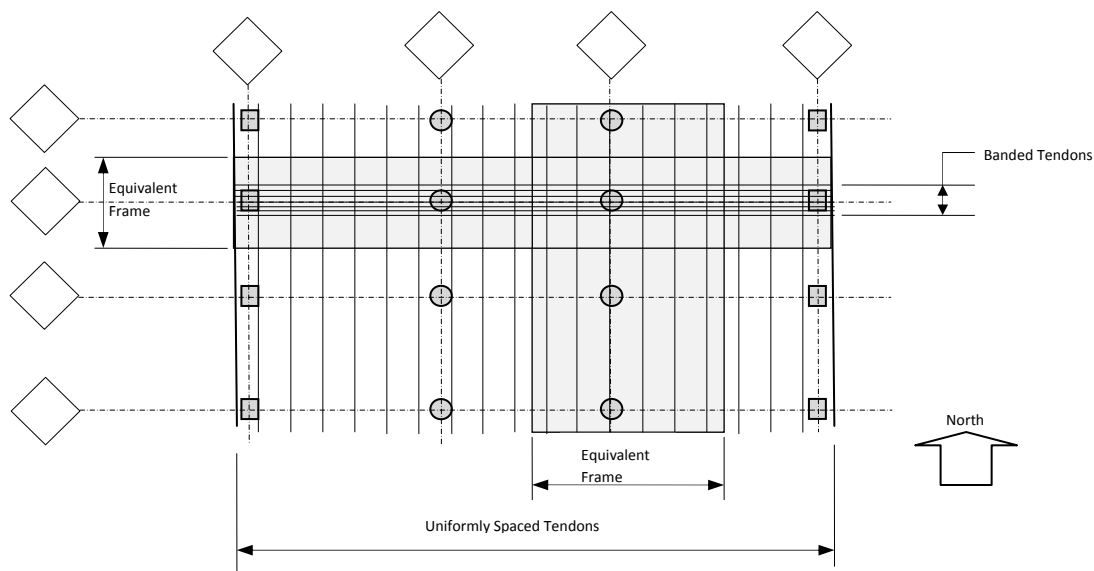
Partial Floor Framing Plan

In the above figure, the banded tendons are grouped in an east-west direction on the columns lines. This three-span slab is analyzed and designed as an equivalent frame. According to ACI 318, each equivalent frame consists of a row of columns or other supports bonded laterally by the centerline of the bay on each side. Therefore, this three-dimensional structure may be simplified to a two-dimensional frame. The stiffness of the columns (or other supports) is usually taken into account when performing the analysis to capture moments resisted by the columns. In the same way, the north-south spanning slab can be analyzed and designed as an equivalent frame.

Note that the EFM is very adaptable to various structural configurations. This method can handle beams on the column lines, non-parallel columns lines, openings in the slab, drop panels and capitols, offset columns, etc. As long as the correct section properties and tributary loads are appropriately modeled, this method will yield accurate results. These non-typical situations, as well as more complex arrangements, are best handled by computer software.
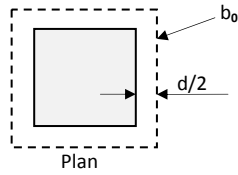
Punching Shear

Another important consideration when designing two-way slab systems, whether conventionally reinforced or post-tensioned, is punching shear. Punching shear, or two-way shear, at column supports is often the controlling factor in establishing a slab thickness. This failure mechanism occurs when the column below the slab literally "punches through" the slab due to overload, insufficient shear strength, or construction defects. This type of failure tends to be catastrophic and therefore the shear strength of the slab at column supports must be examined very carefully. ACI 318 also requires that one-way shear, or beam shear, be checked, but this condition does not usually control in two-way slabs and will not be covered here in this course. ACI 318 also requires, in most situations, two structural integrity tendons to pass through the column core in each direction in case of a punching shear failure. These integrity tendons are intended to prevent a total collapse or a progressive collapse.

There are various techniques to provide adequate punching shear strength in two-way slabs. These include sufficient flat plate thickness, drop panels, column capitols, reinforcing bars, shearhead reinforcement, and headed stud shear reinforcement. In this course, we will only be studying flat plate slabs (slabs without drop panels) without shear reinforcement. The basic concepts used in this course can generally be used to understand the shear reinforcing techniques available to the designer. For example, the use of a drop panel merely increases the slab thickness at the column, providing more shear strength at the column, but also creates a second critical section just outside the drop panel that must also be checked. (Drop panels also influence the equivalent frame stiffness and the distribution of moments and shears). Headed stud shear reinforcement is quite commonly used and is usually included in computer design software.

The first step in analyzing two-way shear in slabs is to define the critical section. The critical section is established by an imaginary line around the slab support along which the support is assumed to punch through the slab. ACI 318 defines the critical section to be located so that its perimeter, b_0, is a minimum but need not be closer to the face of the support than d/2, where d is the depth to the centroid of the tension reinforcing. Thus the critical section is b_0 times d. Note that the critical section is an area over which the shear stress is assumed to be distributed. The dashed line in the following sketches illustrate the location of the critical section for various conditions.
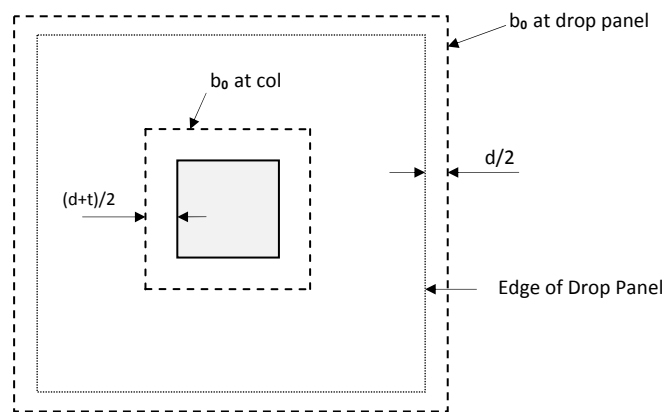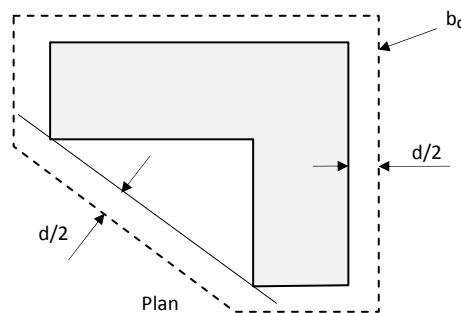
Square Column
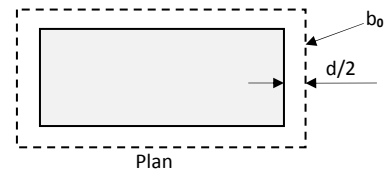
Rectangular Column

Column with Drop Panel

L-Shaped Column

Similar configurations of the location of the critical section are dictated by ACI 318 for columns with punching shear reinforcing consisting of reinforcing bars, steel shear heads, headed stud reinforcing, and the like, but we will only be studying flat plates without shear reinforcing in this course.

It should also be mentioned that openings through the slab in the vicinity of the column tends to reduce the punching shear strength of the slab. They also can reduce the flexural capacity of the slab in one or both directions. ACI 318 gives requirements to account for these openings, but they are beyond the scope of this course.

At columns of two-way pre-stressed slabs, the nominal punching shear strength of the slab is calculated using the following equation:

$$V_c = (\beta_p \lambda \sqrt{f_c'} + 0.3 f_{pc}) b_0 d + V_p$$

Where:

$f_c' \leq 5000$ psi

β_p = the smaller of 3.5 or $(\frac{\alpha_s d}{b_0} + 1.5)$

α_s = 40, 30, and 20 for interior, edge, and corner columns, respectively

λ = Lightweight Concrete Factor, 1.0 for Normal Weight Concrete

125 psi $\leq f_{pc} \leq$ 500 psi, and is the average value in two directions

V_p = vertical component of all effective pre-stress forces

In typical buildings with post-tensioned two-way slabs, normal weight concrete is customarily used and the vertical component of the pre-stress force can be ignored in preliminary design since it is small due to the relatively flat drapes in two-way slabs. However, in real structures where reverse parabolic tendon drapes are normally used, the vertical component of the pre-stress should be included. This is accounted for is computer software packages. Assuming normal weight concrete and neglecting the vertical component of the pre-stress, the nominal punching shear strength of two-way prestressed slabs at columns in typical situations can be simplified to:

$$V_c = (\beta_p \sqrt{f_c'} + 0.3 f_{pc}) b_0 d$$

The above equation applies at interior columns of two-way post-tensioned slabs, and at perimeter columns <u>only if</u> the slab edge extends past the face of the column by at least 4 times the slab thickness h. If the slab does not extend far enough past the face of the column, the pre-stress force in the slab, f_{pc}, is not fully effective, in which case the

nominal shear strength calculation reverts back to the smallest of the following non-pre-stressed two-way nominal shear strength equations:

$$V_c = (2 + \frac{4}{\beta})\lambda\sqrt{f_c'}b_0 d$$

$$V_c = (\frac{\alpha_s d}{b_0} + 2)\lambda\sqrt{f_c'}b_0 d$$

$$V_c = 4\lambda\sqrt{f_c'}b_0 d$$

By examining the above nominal shear strength equations, we can see that the nominal shear strength of the concrete, V_c, is essentially a function of a factor times $\sqrt{f_c'}b_0 d$. The factor depends primarily on the column aspect ratio and whether the column is interior, edge, or corner. It also depends on the term $\frac{\alpha_s d}{b_0}$. This term will be smaller for thinner slabs and larger supports, and could control the maximum punching shear strength.

In most typical buildings, the slab does not usually extend 4h past the outside face of the columns and so the above equations for non-prestressed slabs would apply. Let's say for example that we have an eight-inch thick post-tensioned slab. The edge of the slab would have to project at least 2'-8" past the outer face of support in order to assume the pre-stress force in the slab is fully effective and to be able to use the shear strength equation for prestressed slabs. This amount of slab extension is not usually the case, unless there is a balcony or some other architectural feature. It is usually conservative to use the punching shear equations for non-prestressed slabs for exterior conditions.

Transfer of Moments at Columns

In two-way continuous slab systems, slab moments must be transferred to the column support through the slab-column joint. Slab moments to be transferred may be caused by lateral and/or gravity loads. A portion of the moment is required to be transferred from the slab to the column through flexure, and the remainder is required to be transferred by "eccentricity of shear." This concept is illustrated below.

Edge Column   Interior Column
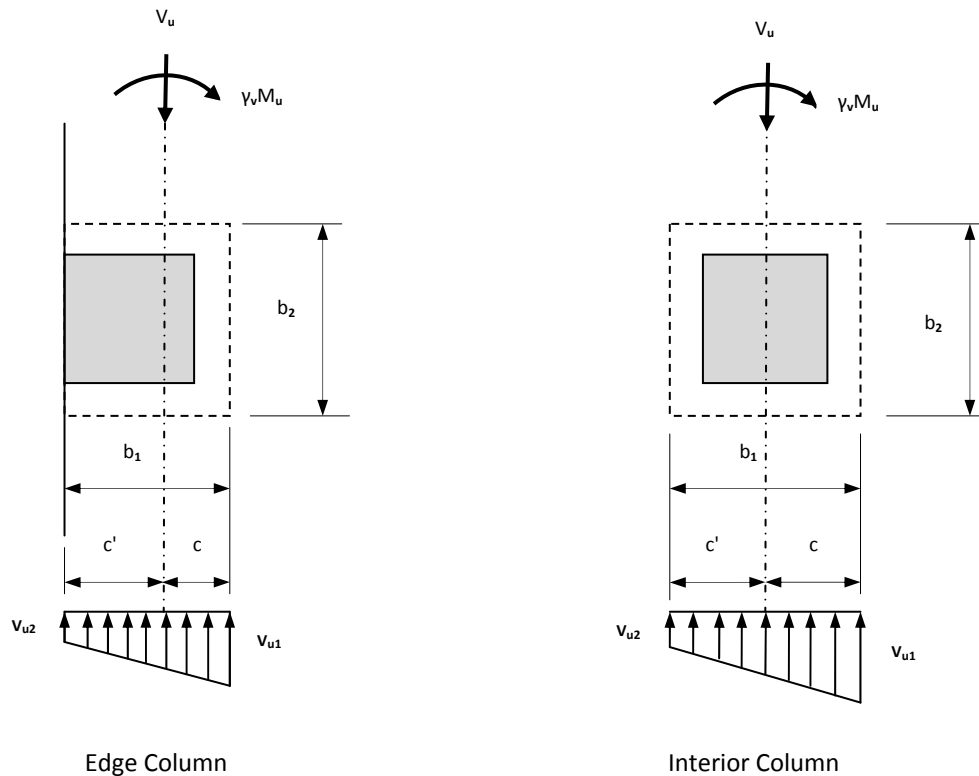
From the above figures, by statics, we can write the following equations for the unit shear stress, v_{u1} and v_{u2}, at the critical section:

$$v_{u1} = \frac{V_U}{A_c} + \frac{\gamma_v M_U c}{J}$$

$$v_{u2} = \frac{V_U}{A_c} - \frac{\gamma_v M_U c'}{J}$$

The above equations are analogous to P/A +/- Mc/I. The total shear demand on the critical section is thus the sum of the direct shear (V_U/A_c) and the eccentricity of shear ($\gamma_v M_U c/J$) due to moment. The terms in the above expressions are defined as:

A_c = The area of concrete resisting shear, or b_0 times the effective depth d, or the critical section

J = A property of the critical section analogous to the polar moment of inertia. Notes on ACI gives a very convenient tabular summary of the section properties of critical sections for rectangular and circular columns.

γ_v = fraction of moment to be transferred by eccentricity of shear = $1 - \gamma_f$

γ_f = fraction of moment to be transferred by flexure = $\gamma_f = \dfrac{1}{1+\left(\frac{2}{3}\right)\sqrt{b_1/b_2}}$

One study found that, for square columns, approximately 60% of the moment is transferred by flexure about the critical section and approximately 40% of the moment is transferred by eccentricity of shear about the centroid of the critical section. ACI provides equations to calculate the fraction of moment transferred by flexure and shear for column shapes other than square. These equations are conveniently graphed in Notes on ACI and, in general, are not very sensitive for aspect ratios up to about 2.0. For example, for a column with an aspect ratio of 2.0, approximately 52% of the moment is transferred by flexure and approximately 48% of the moment is transferred by eccentricity of shear. Therefore, for columns that are square, or very nearly square, a simplified approach would be to use 60% and 40% for moment and shear, respectively. A conservative approach would be to assume 60% for both moment and shear.

For rectangular columns, the portion of the moment that is transferred by eccentricity of shear increases as the dimension of the column *in the direction of the analysis* increases. So for example, for a 48" x 12" column, with the 48" dimension parallel to the direction of the analysis, γ_v = 0.57.

Punching Shear Example with Moment Transfer

Find:

Determine the allowable shear strength of a two-way slab and compare it to the shear demand, considering direct shear and moment. The column is an edge column and the moment is acting perpendicular to the slab edge.
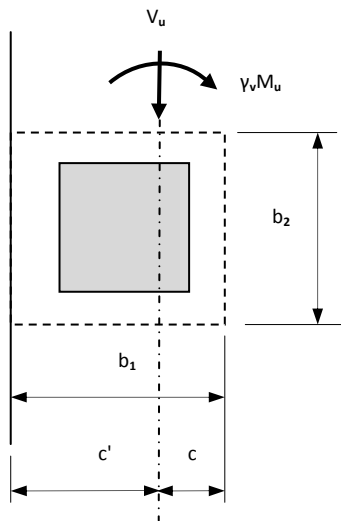
Given:

o Two-Way, Post-Tensioned Flat Plate
o Effective Pre-stress = 165 psi in both directions
o Factored Shear Force V_u = 50 kips
o Factored Moment = 80 foot-kips
o 18" x 18" column
o f'_c = 5,000 psi, normal weight

o Slab h = 7.5 inches (d = 6.5 inches)
o Slab edge extends 6" past outside face of column

Solution:

From the above figure:

b_1 = 6 + 18 + 6.5/2	= 27.25 inches
b_2 = 18 + 6.5	= 24.5 inches
b_0 = 2(27.25) + 24.5	= 79.0 inches

Thus, the area of the critical section is:

A_c = b_0 x d = 79 x 6.5   = 513.5 in^2

From Notes on ACI, for a rectangular edge column with the moment acting perpendicular to the edge, we find the following equations for the properties of the critical section:

$$c = \frac{b_1^2}{2b_1 + b_2} = 9.40 \; inches$$

$$c' = b_1 - c = 17.85 \; inches$$

$$\frac{J}{c} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 4635 \; in^3$$

Since we have a square column, we will transfer 40% of the moment by eccentricity of shear.

$$v_{u1} = \frac{V_U}{A_c} + \frac{\gamma_v M_U c}{J} = \frac{50(1000)}{513.5} + \frac{(0.4)80(12,000)}{4635} = 180 \; psi$$

Now let's compute the shear capacity of the critical section in terms of unit stress. Since the slab edge does not extend past the face of the column by at least 4h, the effective pre-stress force in the slab does not contribute much to the punching shear strength and will be ignored. By inspection of the three equations on page 9 for the nominal punching shear strength of non pre-stressed two-way slabs, we can see that the following equation will be the smallest:

$$V_c = 4\sqrt{f_c'}\, b_0 d$$

In terms of unit shear stress, using the strength reduction factor of 0.75, we compute the shear capacity to be:

$$\phi V_c / b_0 d = \phi 4\sqrt{f_c'}$$

$$\phi v_c = 0.75(4)\sqrt{5000} = 210 \; psi \; > \; 180 \; psi \qquad \textbf{OK}$$

Now we must transfer 60% of the moment by flexure between the slab and the column. The moment to be transferred is 0.60 x 80 ft-kips = 48 ft-kips. According to ACI, this moment transfer must occur in a slab width equal to the column face dimension plus 1.5h on both sides of the column. In this example, the effective slab width is 18 = 2(1.5 x 7.5) = 40.5 inches.

$$R_u = \frac{M_u}{\phi b d^2} = \frac{48(12,000)}{0.90(40.5)(6.5)^2} = 374 \; psi$$

$$\rho = \frac{0.85 f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2R_u}{0.85 f_c'}}\right] = 0.00662$$

$$A_s = \rho b d = 0.00662(40.5)(6.5) = 1.74 \; in^2$$

Thus, we need to make sure we have at least 4#6 bars in the 40.5-inch width in the top of the slab at the exterior column to transfer a portion of the slab moment through the joint into the columns above and below. Note that we have neglected the contribution of the effective pre-stress force to the flexural capacity of the slab section at the column, which is conservative.

Two-Way Banded Flat Plate Design Example

Find:

For the typical hotel floor, determine the required slab thickness and choose an effective post-tensioning force, F_e, in both directions. Also determine the mild reinforcing required. Assume shear walls resist all lateral loads and slabs and columns only carry gravity loads.
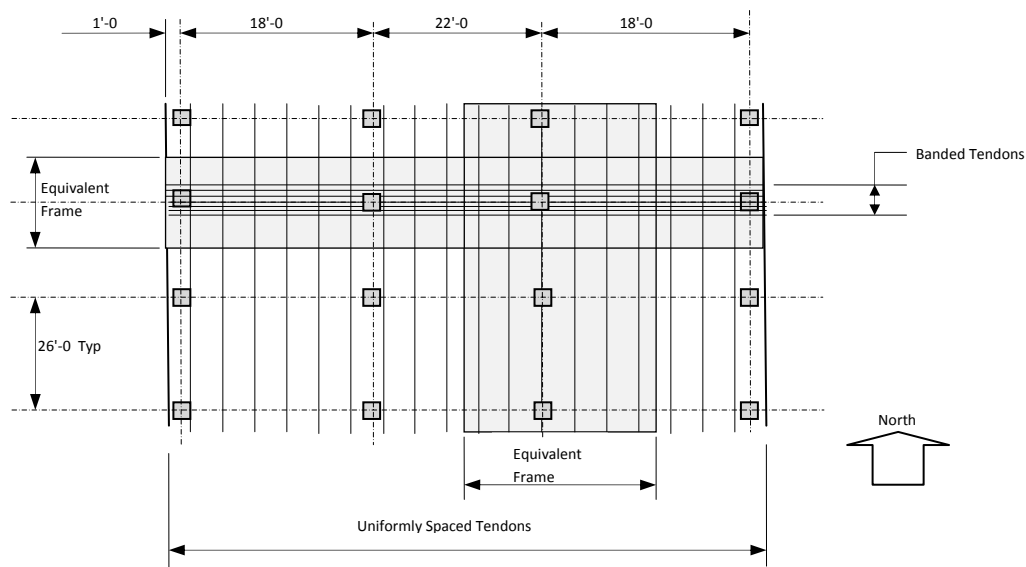
Given:

o f_{PU} = 270 ksi, ½"ϕ unbonded tendons, f_{SE} = 160 ksi
o F_{SE} = 0.153 x 160 = 24.5 kips/tendon
o f'_c = 5,000 psi, normal weight concrete (150 pcf), f'_{ci} = 0.75 f'_c = 3,750 psi
o $E_c = 57\sqrt{f'_c} = 57\sqrt{5000}$ = 3990 ksi
o   Live Load w_L = 40 psf
o w_{BAL} = a minimum of 65% w_D

- o Additional 20 psf superimposed dead load assumed for partitions
- o Column size 18"x18"
- o Story height 10'-0
- o Clear cover to tendons ¾" top and bottom

Solution:

First, let's determine a trial slab thickness. Since we have continuous spans in both directions, we will use the ACI limit on the span-to-thickness ratio of 42 for two-way post-tensioned floor slabs (48 for two-way roof slabs). By inspection, the 26-foot span will control. Thus:

$$h_{min} = \frac{26(12)}{42} = 7.43 \; inches \qquad \boldsymbol{Try \; 7 \, 1/2" \; Slab}$$

The minimum balanced load equals 0.65 x (7.5/12)150 ~ 60 psf

Before we go any further, let's check the punching shear capacity of the trial slab thickness. Let's use a d = 6.5" (3/4" clear and #4 bottom bars). Thus, b_0 = 4(18+6.5) = 98 inches at an interior column.

$$V_c = (\beta_p \sqrt{f_c'} + 0.3 f_{pc}) b_0 d$$

β_p is the smaller of 3.5 or $(\frac{\alpha_s d}{b_0} + 1.5)$, and therefore β_p = 3.5 for interior and exterior columns. And if we use the minimum required effective pre-stress of 125 psi, and using ϕ = 0.75 for shear, we have:

$$\phi V_c = \frac{0.75 \big(3.5\sqrt{5000} + 0.3[125]\big)(98)(6.5)}{1000} = 136 \; kips$$

$$V_u = \frac{[1.2(94 + 20) + 1.6(40)](20)(26)}{1000} = 104 \; kips$$

$$V_u \; \leq \; \phi V_c \qquad \boldsymbol{OK}$$

We will check the shear stresses at the critical section later due to direct shear plus the fraction of moment transferred by eccentricity of shear. Based on observation and experience, there seems to be an adequate margin of strength (136 kips vs. 104 kips) such that there should be enough shear capacity when we include the transfer of moment by eccentricity of shear in the shear stress check later. Also note that although

the b_0 for an exterior column is less than 98 inches, so is the tributary area and shear demand. Again, there seems to be an adequate margin.

Now we can proceed with the equivalent frame analysis using a 7 ½" slab. Let's start with the equivalent frame across the short direction of the building. The width of this equivalent frame will be 26'-0". It has spans of 18', 22', and 18'. According to ACI, if the uniform service live load does not exceed 75% of the uniform service dead load, then all spans may be loaded with the full live load to determine the maximum negative and positive moments. This load condition applies to this example (40 < 0.75 x 114). Furthermore, if we choose to analyze a single floor level within a building, we may consider the far ends of the columns above and below the slab to be fixed. Also note that we have used the full 40 psf live load in this analysis, when it probably could be reduced using live load reductions per the governing Building Code.

Note that the equivalent frame used in this course is an approximation of the more detailed and accurate method described in ACI 318. In ACI 318, an equivalent frame for a two-way slab system is defined with slabs (or beams) spanning between supports that have an increased stiffness from the face of support to the support centerline. Thus, slabs are analyzed with one stiffness (E_cI_g) in the clear span, and a different larger stiffness over the support. Columns are defined similarly, and they have one stiffness (E_cI_g) in the clear height from top of slab to underside of structure (column capitol, bottom of beam, bottom of slab), and an infinite stiffness within the joint. Drop panels and column capitols add another stiffness region near the columns. ACI also defines torsional members, framing into sides of the column perpendicular to the direction of the span being analyzed, that increase the column stiffness in the frame analysis. These requirements are suitable for computer analysis, but we are striving for simplified techniques in this study course.

The procedure used in this course is a simplified version of the equivalent frame method, and we will simply use the gross properties of the members, calculate moments at the centerlines of supports, and assume the far ends of the columns are fixed. Thus, the east-west interior equivalent frame is shown below.
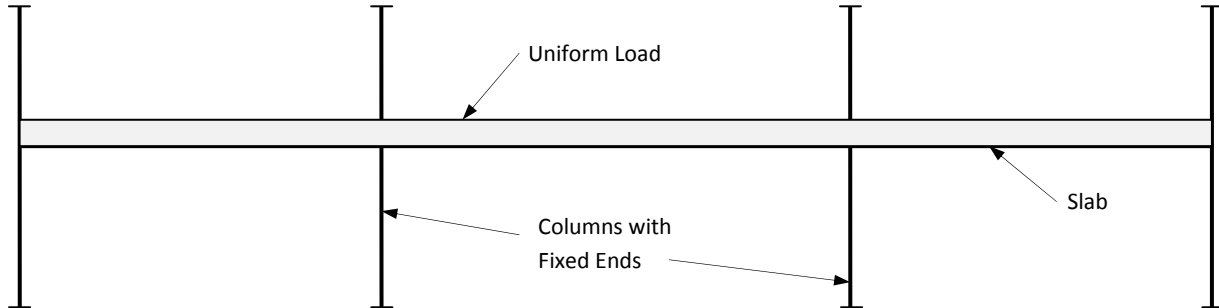
Simplified Equivalent Frame Elevation

Next, we will determine the required effective pre-stress force to balance 65% of the dead load. We will choose to let the uniformly spaced tendons in the orthogonal direction occupy the top layer at the columns, and therefore the d_p for the banded tendons at the column is 7.5 – ¾ - ½ - ¼ = 6". The below figure presents the maximum available drapes in the outer and interior spans. We have 3 7/8" in the outer spans and 5" in the center span.
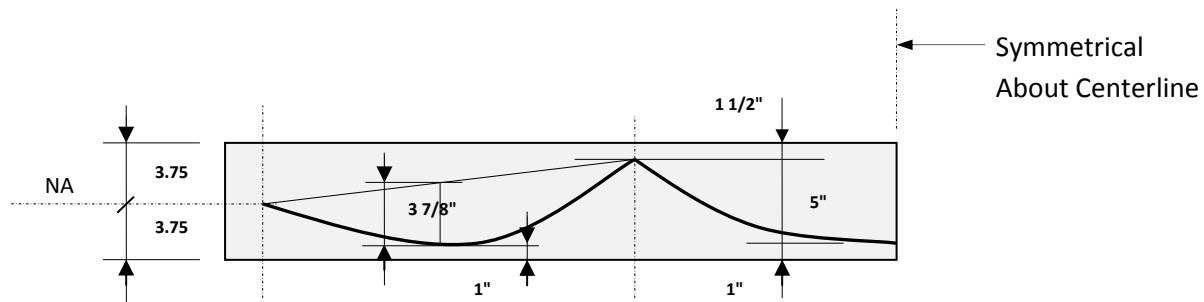
Maximum Available Drapes

The minimum average pre-stress for slabs is 125 psi. So for this 7 ½" slab, our effective pre-stress force must be at least 7.5 x 12 x 0.125 = 11.25 kips/foot/

$$End\ Span\ F_e \ = \ \frac{wL^2}{8a} = \ \frac{0.060(18)^2}{8(3.875)}(12) = 7.5\frac{k}{ft} < 11.25 \qquad \textbf{\textit{NO GOOD}}$$

$$Center\ Span\ F_e \ = \ \frac{wL^2}{8a} = \ \frac{0.060(22)^2}{8(5)}(12) = 8.7\frac{k}{ft} < 11.25 \quad \textbf{\textit{NO GOOD}}$$

Since we need at least 11.25 kips per foot to meet the minimum average pre-stress requirement, we need to balance more than 65% of the dead load. Let's try:

$$\frac{wL^2}{8a} < 11.25 = \frac{w(22)^2}{8(5)}(12)$$

$$w = 0.0774\frac{k}{ft} \quad \Rightarrow \quad 82\% \; w_D$$

Let's use w_{BAL} = 85% w_D = 0.85 x 94 ~ 80 psf

$$End \; Span \; F_e \; = \; \frac{wL^2}{8a} = \frac{0.080(18)^2}{8(3.875)}(12) = 10.0\frac{k}{ft} < 11.25 \quad \textit{NO GOOD}$$

$$Center \; Span \; F_e \; = \; \frac{wL^2}{8a} = \frac{0.080(22)^2}{8(5)}(12) = 11.6\frac{k}{ft} > 11.25 \quad \textit{OK}$$

We have taken advantage of the maximum available drape in the middle span, but we need to adjust the drape in the end spans since we haven't met the minimum average pre-stress requirement in the end spans. We will choose to carry the same balanced load on all three spans for convenience, even though it is not prohibited to carry a different balanced load in different spans. Using a balanced load of 80 psf, we re-calculate the end span drape to be:

$$End \; Span \; Drape = \; a = \frac{wL^2}{8F_e} = \frac{0.080(18)^2}{8(11.6)}(12) = 3.35" \quad \textit{Use 3 3/8"}$$

Thus, we will have a total effective pre-stress force for the banded tendons in all three spans of 11.6 x 26 = 302 kips. For brevity, the analysis for dead, live, and balanced loads has been carried out using 2D software, using gross properties of the 26-foot wide equivalent frame slab with the 18x18 columns above and below. Also, it is assumed that the moment diagrams can be imagined by the reader and so they will not be illustrated. The final slab service moments, in foot-kips, at the column centerlines are as follows:

Service Load Moments	Exterior Column	Exterior Midspan	First Interior Column	Center Midspan
M_{DL}	-65.9	+42.0	-117.4	+61.7
M_{LL}	-23.2	+14.8	-41.2	+21.7
M_{BAL}	+35.4	-22.6	+63.1	-33.1

Note that the above moments could be reduced to the face of the supports. For simplicity, we will use centerline moments in all calculations in this example.

Similar to the one-way slab design example in Part Two of this course, we can tabulate the concrete transfer stresses as follows:

Transfer Stresses	Exterior Column	Exterior Midspan	First Interior Column	Center Midspan
$M_{DL} + M_{BAL}$	-30.5 ft-k	+19.4 ft-k	-54.3 ft-k	+28.6 ft-k
M/S	+/-125 psi	+/-80 psi	+/-223psi	+/-117 psi
P/A	-133 psi	-133 psi	-133 psi	-133 psi
f_t	-8 psi	-53 psi	+90 psi	-16 psi
f_c	-258 psi	-213 psi	-356 psi	-250 psi

The allowable concrete stresses at transfer are:

$f_t \le 3\sqrt{f'_{ci}} = 3\sqrt{3750} = 184$ psi tension

$f_c \le 0.60\ f'_{ci} = 0.6(3750) = 2250$ psi compression

From the above table, the maximum tension and compression values are in the shaded boxes (+90 psi and -356 psi, respectively), and we can see these stresses are well below the allowable values and therefore the stresses at transfer are acceptable.

Now let's check the concrete stresses at service loads (live load, dead load, superimposed dead load, and pre-stress). In this example, our service loads will include sustained dead load of 20 psf for partitions. We will account for this sustained dead load by increasing the dead load moments by the factor of (94+20)/94 = 1.21. Thus, we will algebraically combine M_{SDL}, M_{LL}, and M_{BAL} at the critical sections and compute the concrete stresses, using uncracked section properties.

Service Load Stresses	Exterior Column	Exterior Midspan	First Interior Column	Center Midspan
$M_{SDL} + M_{LL} + M_{BAL}$	-67.5 ft-k	+13.4 ft-k	-120.2 ft-k	+63.3 ft-k
M/S	+/-277 psi	+/-55 psi	+/-493psi	+/-260 psi
P/A	-133 psi	-133 psi	-133 psi	-133 psi
f_t	+144 psi	-78 psi	+360 psi	+127 psi
f_c	-410 psi	-188 psi	-626 psi	-393 psi

ACI requires that two-way pre-stressed slab systems be considered as Class U, or uncracked, and the maximum permissible service load concrete tensile stress is:

$$f_{tens} \le 6\sqrt{f'_c} = 6\sqrt{5000} = 420\ psi$$

And the extreme fiber stress in compression is:

$$f_{comp} \leq 0.45 f_c' = 0.45(5000) = 2250 \ psi$$

From the above table, the maximum tension and compression values are in the shaded boxes (+360 psi and -626 psi, respectively), and we can see these stresses are well below the allowable values and therefore the stresses at service loads are acceptable.

Next, let's determine the hyperstatic moments, $M_{HYP} = M_{BAL} - M_1$. Recall that M_1 is the effective post-tensioning force times the eccentricity from the neutral axis of the member at critical locations. Using F_e = 302 kips, we can tabulate the hyperstatic moments as follows:

	Exterior Column	Exterior Midspan	First Interior Column	Center Midspan
M_{BAL}	+35.4 ft-k	-22.6 ft-k	+63.1 ft-k	-33.1 ft-k
e	0"	-2.75"	+2.25"	-2.75"
M_1 ($F_e$xe)	0 ft-k	-69.2 ft-k	+56.6 ft-k	-69.2 ft-k
M_{HYP}	+35.4 ft-k	+46.6 ft-k	+6.5 ft-k	+36.1 ft-k

Now that we have computed all the bending moment demands, and have confirmed that service load stresses are OK, we can proceed with determining the factored moments. According to ACI 318, the factored moment is calculated using:

$$M_u = 1.2 \ M_{DL} + 1.6 \ M_{LL} + 1.0 \ M_{HYP}$$

The factored moments, M_U, are tabulated as follows:

	Exterior Column	Exterior Midspan	First Interior Column	Center Midspan
M_{UDL}	-79.1 ft-k	+50.4 ft-k	-140.1 ft-k	+74.0 ft-k
M_{ULL}	-37.2 ft-k	+23.7 ft-k	-65.9 ft-k	+34.7 ft-k
M_{HYP}	+35.4 ft-k	+46.6 ft-k	+6.5 ft-k	+36.1 ft-k
M_U	-80.9 ft-k	+120.7 ft-k	-199.5 ft-k	+144.8 ft-k

Now that we understand the factored moment demand for the floor slab in this direction, our next task is to determine the quantity of bonded mild reinforcing steel is required to satisfy the demand. First, let's discuss the ACI 318 minimum requirements for two-way post-tensioned flat slab systems.

ACI 318 does not require bonded reinforcement in positive moment areas (tension in the bottom of the slab) if the service load tensile stress $f_t \leq 2\sqrt{f_c'}$. for 5,000 psi concrete in this example, this limit equals +140 psi. (Note that the maximum concrete compressive strength for two-way post-tensioned slab systems is 5,000 psi and the maximum value for $\sqrt{f_c'}$ is 70 psi). The maximum tensile stresses in positive moment areas in this example are -78 psi and +127 psi for Exterior Midspan and Center Midspan, respectively, which are both less than the +140 psi limit. Thus, minimum bonded steel is not required in positive moment areas in this direction of analysis.

In negative moment areas (tension in the top of the slab), the minimum area of bonded reinforcing in each direction is:

$$A_s = 0.00075 A_{cf}$$

Where A_{cf} is the larger gross cross-sectional area of the equivalent frames in two orthogonal directions at the column. This reinforcement shall be distributed between lines that are 1.5h outside opposite sides of the column, be at least four bars, and be spaced no more than 12" on center. In this example, the larger equivalent frame is the one we are currently designing, and is 26 feet wide. Thus,

$$A_s = 0.00075 A_{cf} = 0.00075(12)(26)(7.5) = 1.76 \, in^2$$

Bars must be distributed over a width of 1.5 x 7.5 x 2 + 18 = 40.5 inches. With the maximum spacing of 12", we need at least 5 bars. USE: 6#5 Top both ways.

Now we can check the ultimate flexural capacity using the minimum area of bonded mild reinforcing steel. The span-depth ratio is 18x12/7.5 = 29 which is less than 35. The span-depth ratio for the 22-foot span equals 35. Therefore, the following equation (refer to Part One of this course) applies in order to determine the stress in the pre-stressing steel at nominal flexural capacity:

$$f_{ps} = f_{se} + 10,000 + \frac{f_c'}{100\rho_p} = 160,000 + 10,000 + \frac{5,000}{100\,\rho_p} =$$

$$\leq f_{py} \ or \ f_{se} + 60,000$$

The ratio of pre-stressed reinforcement, ρ_p, is the same for all spans and on a per foot basis is:

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{(0.153)}{(12)(6)} \times \frac{11.6 \, k/ft}{24.5 \, k/tendon} = 0.00101$$

$$f_{ps} = f_{se} + 10,000 + \frac{5,000}{100 \, \rho_p} = 170,000 + \frac{5,000}{100(0.00101)} = 220 \, ksi$$

This value for f$_{ps}$ is less than f$_{PY}$ = 270 ksi and equal to f$_{SE}$ + 60,000 = 220 ksi and therefore f$_{ps}$ = 220 ksi.

Now we can calculate the nominal moment capacity. The depth of the bonded reinforcing steel, if any, is at the same depth as the tendons. The area of unbonded tendons in the full band width of 26 feet is:

$$A_{ps} = \frac{(0.153)\left(11.6 \frac{k}{ft}\right)(26)}{24.5} = 1.88 \, in^2$$

In negative moment areas, with 6#5 top bars, and using the full band width of 26x12 = 312 inches:

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{(1.88)(220) + (6)(0.31)(60)}{(0.85)(5)(312)} = 0.396 \, inches$$

$$\phi M_n = 0.90[(1.88)(220) + (6)(0.31)(60)]\left(6 - \frac{0.396}{2}\right)\left(\frac{1}{12}\right)$$

$$\phi M_n = 229 \, ft - k > 199.5 \, ft - k \quad \textbf{OK}$$

In positive moment areas, with no mild reinforcing:

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{(1.88)(220) + 0}{(0.85)(5)(312)} = 0.312 \, inches$$

$$\phi M_n = 0.90[(1.88)(220)]\left(6.5 - \frac{0.312}{2}\right)\left(\frac{1}{12}\right)$$

$$\phi M_n = 197 \, ft - k > 144.8 \, ft - k \quad \textbf{OK}$$

Therefore, the nominal moment capacity computed with minimum bonded reinforcing steel in negative moment areas, and without bonded reinforcing steel in positive moment areas, is greater than the demand and is acceptable.

We must now investigate the shear stress in the slab at the columns in the direction of the analysis. Let's begin by determining the critical section, and then calculate the nominal shear capacity using the appropriate equation. After that, we will calculate the shear stress demand.

For the exterior column, we can see that the slab does not extend at least 4h past the face of the columns, and therefore we cannot take advantage of the post-tensioning to calculate the shear capacity and must revert to the non pre-stressed equation:

$$\phi v_c = \phi 4\sqrt{f_c'} = (0.75)(4)\sqrt{5000} = 210 \; psi$$

To calculate the punching shear demand, we need to determine the factored shear forces and the transfer moments at each column. The factored moments are the algebraic sum of the moments due to external loading (sustained dead loads, live loads, etc.) plus the hyperstatic moments. The load factor combination for moment is $1.2M_D + 1.6M_L + 1.0M_{HYP}$. For the exterior column, assuming we have a uniform exterior wall load of 25 psf x 10 ft = 250 plf, we have:

V_U = [1.2(94+20) + 1.6(40)](10x26) + 1.2 x 250 x 26 = 60 k

And from the table above, M_U = -80.9 ft-k

Since we have a square column, we will transfer 40% of the unbalanced moment by eccentricity of shear. The exterior b_0 = 2(3+18+6.5/2) + (18+6.5) = 73 inches, and using the equations in ACI Notes we find J/c = 3986, and J/c' = 1983.

$$v_{u1} = \frac{V_U}{A_c} + \frac{\gamma_v M_U c}{J} = \frac{60,000}{73.5(6.5)} + \frac{(0.4)80.9(12,000)}{3986} = 223 \; psi$$

$$v_{u2} = \frac{V_U}{A_c} - \frac{\gamma_v M_U c'}{J} \quad \frac{60,000}{73.5(6.5)} - \frac{(0.4)80.9(12,000)}{1983} = -70 \; psi$$

Thus, the shear demand at the exterior column is greater than the allowable shear stress of 210 psi. Therefore, we need to increase the shear capacity of the slab at the exterior column. At this point in the analysis, since the shear stress is a modest 6% over the allowable limit, it is probably most practical and economical to add headed stud reinforcing, which is beyond the scope of this course but is commonly handled by computer software.

For the interior column, β_p = 3.5 < $(\frac{\alpha_s d}{b_0} + 1.5)$ = 4.15, and we have:

$$\phi v_c = \phi(\beta_p\sqrt{f_c'} + 0.3f_{pc}) = 0.75[3.5\sqrt{5000} + 0.3(129)] = 213\ psi$$

We must refer to the 2D computer analysis to find the *unbalanced* moments at the interior column. The unbalanced moment at the interior column is due to the unbalanced dead and live loads and the different adjacent span lengths. This is the moment that must be transferred from the slab to the column by eccentricity of shear. From the 2D analysis, at an interior column, we find:

M_U = 38.6 ft-k

V_U = [1.2(94+20) + 1.6(40)] (20x26)/1000 = 104 k

The interior b_0 = 4(18+6.5) = 98 inches, and using the equations in ACI Notes we find J/c = J/c' = 5294.

$$v_{u1} = \frac{V_U}{A_c} + \frac{\gamma_v M_U c}{J} = \frac{104,000}{98(6)} + \frac{(0.4)38.6(12,000)}{5294} = 212\ psi$$

$$v_{u2} = \frac{V_U}{A_c} - \frac{\gamma_v M_U c'}{J}\ \frac{104,000}{98(6)} - \frac{(0.4)38.6(12,000)}{5294} = 142\ psi$$

Thus, the shear demand at the interior column is less than the allowable shear stress of 213 psi and therefore is acceptable.

The last thing we need to check in this direction of analysis is the unbalanced moment transfer by flexure at the columns. We need to transfer 60% of the unbalanced moment by flexure. Thus, we need to transfer (0.60)(80.9 ft-k) = -48.5 ft-k and (0.60)(38.6 ft-k) = 23.2 ft-k at the exterior and interior columns, respectively. This moment transfer must occur in a slab width equal to the column face dimension plus 1.5h on both sides of the column, or 18 = 2(1.5 x 7.5) = 40.5 inches.

$$R_u = \frac{M_u}{\phi bd^2} = \frac{48.5(12,000)}{0.90(40.5)(6)^2} = 444\ psi$$

$$\rho = \frac{0.85f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2R_u}{0.85f_c'}}\right] = 0.00783$$

$$A_s = \rho bd = 0.00783(40.5)(6) = 1.90 \ in^2$$

Based on this demand, we can see that the minimum area of bonded mild reinforcement of 6#5 is not quite enough. For symmetry, we will choose to use a total of 8#5 bars, two in the column core, and three on both sides of the column in a width of 1.5h = 11 ¼".

To summarize the analysis in this direction, we have used a slab thickness of 7 ½" with an effective pre-stress force of 11.6 kips per foot. So, in the 26-foot wide equivalent frame, we need a total effective pre-stress of 302 kips. Using 24.5 kips per ½"ϕ tendon, we need 13 tendons (each will be stressed slightly less than 24.5 kips) placed in a narrow band on the column line in the east west direction, with at least two of them passing through the column core to act as integrity tendons.

Although there are no specific ACI 318 requirements for the width in which the banded tendons should be placed, tendons should generally not placed closer together than 4d$_b$ on center, and should be placed in as narrow a band as possible to facilitate the placing of the uniformly spaced tendons in the perpendicular direction. In slabs, it is common to bundle two to four tendons together in a plane and then space these bundles as required.

In practical terms, the tendons in a band could be distributed between lines that are 1.5h outside opposite sides of the column, similar to the bonded reinforcing. Thus, in this example, it would be acceptable to place three tendons through the column core and five tendons on both sides of the column at 2" on center, or in a bundle of two and three each. The spacing of individual tendons or bundles of tendons shall not exceed the smaller of 8h or 5 feet, according to ACI.

Now, let's turn our attention briefly to the analysis in the perpendicular, or north-south direction. The spans are 26'-0 each. We essentially have two equivalent frame analysis cases: one case at the perimeter edge of the building and the other case at the interior of the building. Recall that we have selected the d$_P$ in this direction to be 7.5 – 3/4" – 1/4" = 6.5".

Ignoring the end span, we will determine the required effective pre-stress force to balance 65% of the dead load. The below figure presents the maximum available drape in the typical interior span using ¾" clear top and bottom and ½" diam. unbonded tendons.
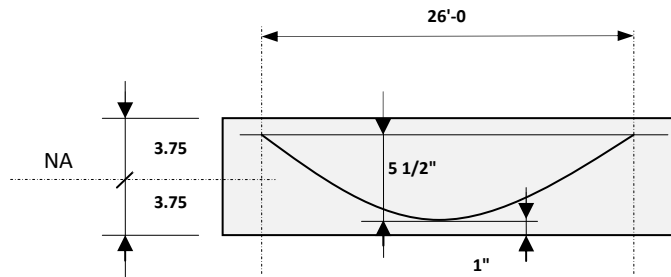
Maximum Available Drape

$$F_e = \frac{wL^2}{8a} = \frac{0.060(26)^2}{8(5.5)}(12) = 11.06\frac{k}{ft} \quad < \quad 11.25\ \frac{k}{ft} \qquad \textbf{\textit{NO GOOD}}$$

The required effective pre-stress force to balance 65% of the dead load is less than the minimum of 125 psi, or 11.25 k/ft, so we will choose to balance a higher percentage of the dead load. Let's balance 70% of the dead load, or 65 psf. Thus, we have:

$$F_e = \frac{wL^2}{8a} = \frac{0.065(26)^2}{8(5.5)}(12) = 12.0\frac{k}{ft} \quad > \quad 11.25\ \frac{k}{ft} \qquad \textbf{\textit{OK}}$$

An equivalent frame analysis was carried out using 2D computer software. Although both the perimeter and interior equivalent frames need to be analyzed, for brevity the results only for the interior frame are tabulated below. The interior equivalent frame is 20'-0 wide. We are ignoring the end span and only show the typical interior continuous span. If the end span in this example were 26'-0, then owing to the fact that there is less available drape in the end span, additional pre-stressing force could be easily added to the end span.

From the 2D analysis:

Service Moments	Column	Midspan
Service M_{DL}	-110.1 ft-k	+52.8 ft-k
Service M_{LL}	-46.9 ft-k	+22.5 ft-k
Service M_{BAL}	+71.4 ft-k	-34.2 ft-k

Transfer Stresses	Column	Midspan
Service M$_{DL}$ + M$_{BAL}$	-38.7 ft-k	+18.6 ft-k
M/S	+/-206 psi	+/-99 psi
P/A	-125 psi	-125 psi
f$_t$	+81 psi	-26 psi
f$_c$	-331 psi	-224 psi

The allowable concrete stresses at transfer are:

f$_t$ ≤ 3 √f'$_{ci}$ = 3 √3750 = 184 psi tension

f$_c$ ≤ 0.60 f'$_{ci}$ = 0.6(3750) = 2250 psi compression

From the above table, the maximum tension and compression values are in the shaded boxes (+81 psi and -331 psi, respectively), and we can see these stresses are well below the allowable values and therefore the stresses at transfer are acceptable.

And the sustained service load stresses are:

Service Load Stresses	Column	Midspan
Service M$_{SDL}$ + M$_{LL}$ + M$_{BAL}$	-109.0 ft-k	+52.3 ft-k
M/S	+/-581 psi	+/-279 psi
P/A	-125 psi	-125 psi
f$_t$	+456 psi	+154 psi
f$_c$	-706 psi	-404 psi

The maximum permissible service load concrete tensile stress is:

$$f_{tens} \leq 6\sqrt{f_c'} = 6\sqrt{5000} = 420 \, psi$$

And the extreme fiber stress in compression is:

$$f_{comp} \leq 0.45 f_c' = 0.45(5000) = 2250 \, psi$$

From the above table, the maximum tension and compression values occur at the column and are in the shaded boxes (+456 psi and -706 psi, respectively). We can see the compressive stress is acceptable, but the tensile stress exceeds the allowable value at the column. The tensile stress at the bottom of the slab at midspan (+154 psi) also exceeds the allowable value of $2\sqrt{f_c'}$ = 140 psi, and therefore requires bonded reinforcing. When the tensile stresses exceed the limit, then the minimum area of bonded reinforcing is:
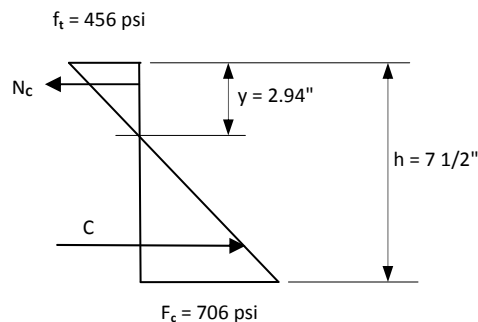
$$A_s = \frac{N_c}{0.5 f_y} \qquad \text{Where } N_c = \text{Total Tension Force}$$

Using the tension stress of 456 and the compression stress of 706 at the column, we find by similar triangles the distance y in the figure below to be 2.94".

f_t = 456 psi

N_c

y = 2.94"

h = 7 1/2"

C

F_c = 706 psi

$$A_s = \frac{(456)(2.94)(12)(20)/2}{0.5(60,000)} = 5.36 \ in^2 \quad (18\#5)$$

This area of steel is to be distributed across the entire equivalent frame width of 20 feet. Although ACI does not specify how this steel should be distributed, it would seem reasonable to place 75% of the total in the "column strip" and 25% in the two half "middle strips" as in the direct design method. Thus, we will place 12#5 in a 10-foot band centered on the column, and 3#5 on both sides outside of this band.

Similarly, in the positive moment areas at midspan, we find the total required area of bonded steel to be 1.28 in² across the entire 20-foot equivalent frame. We will use 14#4 (which is #4@18" o.c.).

After finding the hyperstatic moments (M_{HYP} = M_{BAL} – 12.0 x 20 x e, where e = +2.75" at the column and -2.75" at midspan), the factored moments, M_U, are tabulated as follows:

Factored Moments	Column	Midspan
M_{UDL}	-160.2 ft-k	+76.8 ft-k
M_{ULL}	-75.0 ft-k	+36.0 ft-k
M_{HYP}	+16.4 ft-k	+20.8 ft-k
M_U	-218.8 ft-k	+133.6 ft-k

The span-depth ratio is 26x12/7.5 = 42 which is greater than 35. Therefore, unlike the perpendicular banded direction, the following equation (refer to Part One of this course) applies in order to determine the stress in the pre-stressing steel at nominal flexural capacity:

$$f_{ps} = f_{se} + 10,000 + \frac{f_c'}{300\rho_p} = 160,000 + 10,000 + \frac{5,000}{300\,\rho_p} =$$

$$\leq f_{py} \; or \; f_{se} + 30,000$$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{(0.153)}{(12)(6.5)} \times \frac{12.0 \, k/ft}{24.5 \, k/tendon} = 0.000961$$

$$f_{ps} = f_{se} + 10,000 + \frac{5,000}{300\,\rho_p} = 170,000 + \frac{5,000}{300(0.000961)} = 187 \, ksi$$

This value for f_{ps} is less than f_{PY} = 270 ksi and less than f_{SE} + 30,000 = 190 ksi and therefore f_{ps} = 187 ksi.

For positive moment areas, including #4@18, we find:

$$+\phi M_n = 202 \, ft-k > 133.6 \, ft-k \quad \textbf{OK}$$

For negative moment areas, using just the 12#5 in the column strip, we find:

$$-\phi M_n = 227 \, ft-k < 218.8 \, ft-k \quad \textbf{OK}$$

Checks that remain for this direction of analysis, but will not be shown here for brevity, are the punching shear stress calculations, including the eccentricity of shear, and the moment transfer. Also, note that we have only shown an interior equivalent frame, and understand that an exterior equivalent frame would need to be analyzed as well. Deflection should also be reviewed, but deflection calculations are beyond the scope of this course.

To summarize the analysis in this direction, using a slab thickness of 7 ½" with an effective pre-stress force of 12.0 kips per foot. Using 24.5 kips per ½"ϕ tendon, we need tendons at 24 1/2" on center. At least two of these tendons need to pass through the column core to act as integrity tendons. Most likely, the tendons in this direction would be in bundles of two or three; with a bundle of two passing through the column core, and for symmetry, bundles of two spaced at 49" on center on both sides of the

column centerline. Thus, 10 tendons would be uniformly distributed over the 20-foot width of the equivalent frame. There would also be bonded top bars at each column in this direction, as well as bonded bottom reinforcing at each midspan.
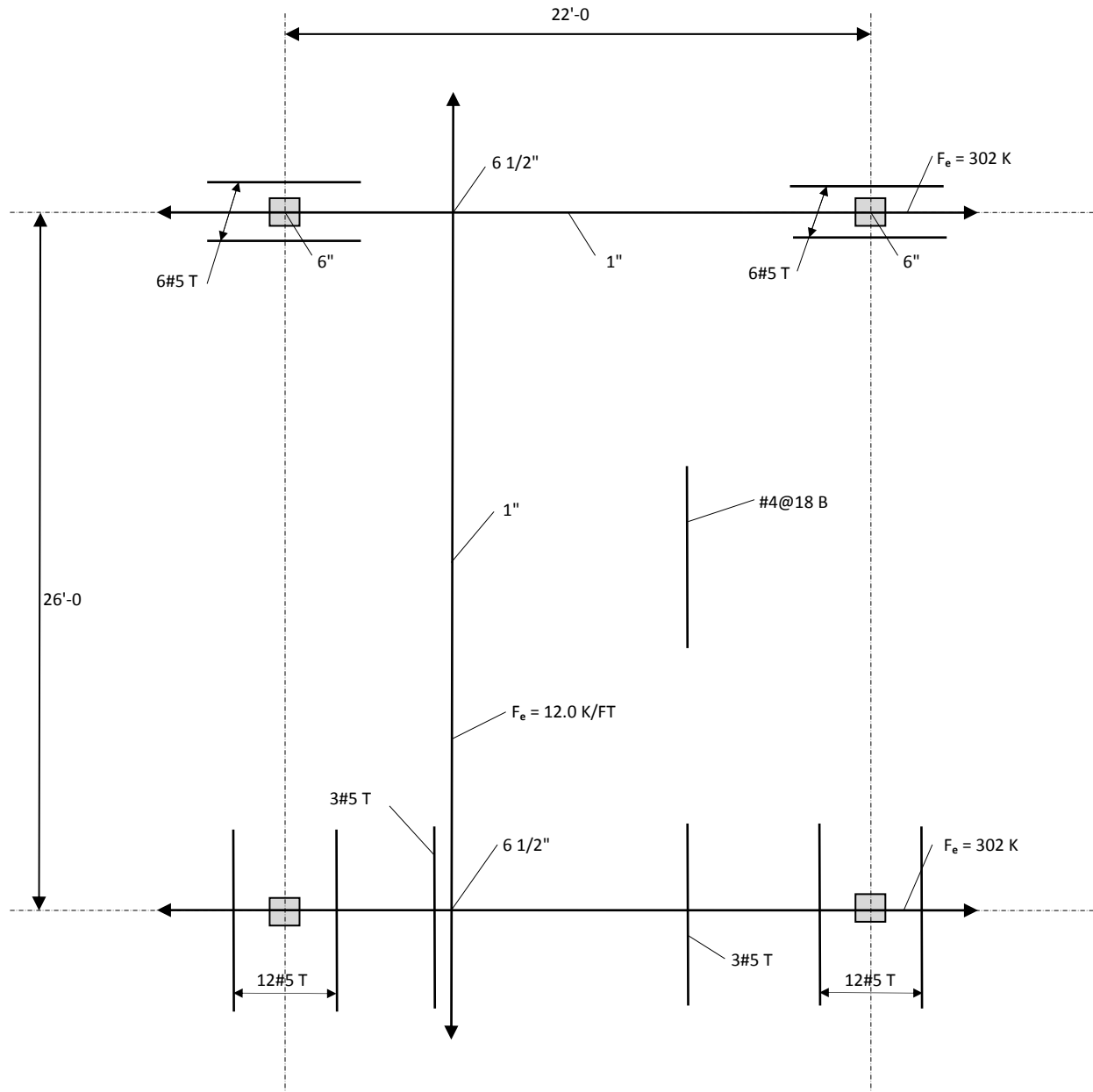
Final Design Sketch of an Interior Bay

Conclusion

Part Three of this three-part course covers one of the more advanced topics related to post-tensioned concrete for building structures using unbonded tendons. It should be abundantly obvious by now that that the analysis and design of post-tensioned systems, especially two-way systems, can be extremely tedious and difficult to accomplish by hand and is best handled by computer software specifically intended for this purpose. However, with a good understanding of the material in Part Three of this course, you should:

- Be able to understand and quickly spot-check a computer generated design for a two-way slab system.
- Understand how to compute an approximate effective pre-stress force using an assumed balance load.
- Review and understand punching shear for a two-way slab, including eccentricity of shear.
- Understand allowable stresses at transfer and service loads according to ACI 318-08.
- Determine the amount of bonded flexural reinforcing and when it is required.
- Calculate the nominal moment capacity ϕM_n and nominal punching shear capacity ϕV_n of a two-way slab system.
- Check the moment transfer capacity of the slab at the support.

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