Soils and Foundations 2012;52(4):575–589

The Japanese Geotechnical Society

Soils and Foundations

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An elastoplastic model for soft sedimentary rock considering inherent anisotropy and confining-stress dependency

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Received 2 November 2011; received in revised form 6 April 2012; accepted 1 May 2012
Available online 7 September 2012

Abstract

The inherent anisotropy of soft sedimentary rock is a very important factor that influences the mechanical behavior of the rock. The confining-stress dependency of the shear stress ratio at the critical state, briefly referred to as the confining-stress dependency, is another important factor that should be taken into consideration when discussing the mechanical behavior of the rock. In this paper, based on an elasto-viscoplastic constitutive model for soft sedimentary rock (Zhang et al., 2005), a new model capable of describing both the inherent anisotropy and the confining-stress dependency of soft sedimentary rock is proposed in the framework of generalized stress space, called the t_{ij} concept (Nakai and Mihara, 1984), and the subloading yield surface (Hashiguchi and Ueno, 1977; Hashiguchi, 1989). In order to describe the confining-stress dependency, an evolution equation for the shear stress ratio at the critical state M^* is introduced. A transformed stress, proposed by Boehler and Sawczuk (1977) and first introduced into a constitutive model by Oka et al. (2002), is also adopted into the model. In order to examine the performance of the newly proposed model, triaxial compression tests for soft sedimentary rock under different loading conditions were simulated and the results have been compared with the corresponding test results obtained from the authors and other researches available in the literature. It is found that the model can describe both the inherent anisotropy and the confining-stress dependency of soft sedimentary rock using a set of parameters with a fixed value for a given material.
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Keywords: Soft sedimentary rock; Elastoplastic model; Inherent anisotropy; Confining-stress dependency; Triaxial test; Transformed stress tensor

1. Introduction

It is known that the mechanical behavior of soft sedimentary rock is elastoplastic, strain-hardening, strain-softening and time dependent. In addition to those, the confining-stress dependency and inherent anisotropy of soft sedimentary rock are also very important factors that should be taken into consideration in modeling soft rock. Many experimental and theoretical investigations have been done on the mechanical behavior of soft sedimentary rock. The test results and the various constitutive models proposed to demonstrate and describe the above-mentioned mechanical features of soft sedimentary rock can be found in the literature.

Adachi and Ogawa (1980) conducted a large number of drained triaxial compression tests on a soft sedimentary rock called Ohya stone, a typical green tuff distributed widely in the Tohoku Area of Japan. In their paper, it is

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Peer review under responsibility of The Japanese Geotechnical Society.

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http://dx.doi.org/10.1016/j.sandf.2012.07.001

reported that shear stress ratio at critical state $R_{CS}=(\sigma_1/\sigma_3)|_{critical}$ is largely dependent on confining-stress and takes different values at different confining stresses. Iwata (2011) also reported the same phenomenon based on a large number of drained triaxial and plane-strain compression tests on Ohya stone. Oka and Adachi (1985) and Adachi and Oka (1995) proposed an elastoplastic model to describe strain softening of soft sedimentary rock using the concept of endochronic theory (Valanis, 1971) and taking the time as internal variables. Zhang et al. (2003) proposed a model to describe the mechanical behavior of the soft rock under true triaxial stress condition by introducing t_{ij}-concept (Nakai and Mihara, 1984) and Matsuoka–Nakai failure criteria into the model proposed by Oka and Adachi (1985). In the work by Iwata (2011), confining-stress dependency was modeled by introducing a proper evolution equation for the change of the shear stress ratio at critical state. As a result, it has become possible to uniquely describe the mechanical behavior of the soft rock under different confining stresses with a set of parameters with fixed values. All the models mentioned above, however, are based on the assumption that the soft rock is an isotropic geomaterial.

It is important to keep in mind that soft sedimentary rock is usually found in stratified masses, which means that the anisotropy of the rock is an important feature of its mechanical behavior. There are two different kinds of anisotropy in geomaterials. One is the inherent anisotropy that formed in its long-term sedimentary process of soft rock, such as Ohya stone. The inherent anisotropy of a geomaterial subjected to external load usually will not change unless the bonding of the material has broken and has reached a failure state. Another anisotropy is the so-called stress-induced anisotropy, which is an important factor that influences the mechanical behaviors of soft soil. Unlike inherent anisotropy, the stress-induced anisotropy may change with plastic deformation during the loading process, and does not require the rock to be in a failure state. In this paper, however, we only focus on the inherent anisotropy of soft rock, and abbreviate it simply as 'anisotropy'.

The anisotropy of soft rock usually can be regarded as a transverse isotropy formed in its sedimentary environment. Some pioneer-research work on the anisotropy of rock can be found in the literature, e.g. the works by Niandou et al. (1997), Oka et al. (2002) and Rouabhi et al. (2007). A series of drained triaxial compression tests on the anisotropic mechanical behavior of soft sedimentary rock have been reported by Kobayashi (2000). From the test results, it is known that the characteristics of the strength and the dilatancy of the soft sedimentary rock are strongly dependent on the orientation of the maximum principal stress towards the preferential layered plane.

In order to properly introduce the anisotropy into the failure criteria for transversely isotropic materials, several mathematical methods were developed by Boehler and Sawczuk (1977), Boehler (1987), Cazacu et al. (1998) and Car et al. (2001), in which Boehler's theory is simple and can be easily extended to an isotropic model considering strain-hardening, strain-softening. Oka et al. (2002) proposed an anisotropic model for Ohya Stone by introducing Boehler's method into the isotropic model proposed by Oka and Adachi (1985).

Zhang et al. (2005) also proposed an elasto-viscoplastic model for soft sedimentary rock, which is capable not only of describing such mechanical features of the soft rock, such as strain-hardening, strain-softening behavior, the stress–dilatancy relation and the time dependency, but also the intermediate-stress dependency that always need to be considered under the true triaxial stress condition. The model is developed based on the concepts of subloading surface (Hashiguchi and Ueno, 1977; Hashiguchi, K., 1989) and t_{ij} (Nakai and Mihara, 1984). The model can describe the behavior of strain softening and takes into account the influence of intermediate principle stress on the strength of geologic materials. In order to establish an anisotropic model that can properly describe anisotropy as well as the other mechanical behavior of the soft sedimentary rock aforementioned, the model proposed by Zhang et al. (2005), which hereafter will be referred to as the original model, should be modified.

In this paper, by introducing Boehler's method for the anisotropy, and the method for the confining-stress dependency proposed by Iwata (2011) into the original model, a new anisotropic model for sedimentary soft rock is developed. The new model can describe all the mechanical features of soft rock, including the strain-hardening and strain-softening behavior, the stress–dilatancy relation, the anisotropy, and the dependency on time, the intermediate-stress and the confining-stress. The performance of the model is confirmed with drained compression tests under different loading conditions.

2. Derivation of anisotropic elastoplastic model

As aforementioned, the anisotropy of soft sedimentary rock is a transverse isotropy that depends on the normal direction of the sedimentation surface. The transversely isotropic plane of the soft sedimentary rock is just the sedimentation plane. Boehler and Sawczuk (1977) proposed an orientation tensor and developed an anisotropic yielding framework for the transversely isotropic materials using the orientation-tensor-based transformed stress. In this paper, based on Boehler's theory, the orientation tensor and the transformed stress are adopted to the isotropic model to describe the transversely isotropic effect of soft sedimentary rock.

In the work by Boehler and Sawczuk (1977), the stress tensor σ_{ij} is replaced by a transformed stress tensor $\hat{\sigma}_{ij}$ in the yield surface and a flow rule, which is the same as that in traditional isotropic constitutive models. The transformed stress tensor can be regarded as a fictional stress tensor in an equivalent isotropic stress space, in which the yield surface and the flow rule of the anisotropic material are defined. As a result, the anisotropic material can be

treated as an equivalent isotropic material in the yield surface and in the flow rule using the concept of the transformed stress.

2.1. Concept of stress transformation

The transformed stress tensor is expressed by a function of the real stress tensor σ_{ij} and an orientation tensor m_{ij} of the sedimentation plane. The orientation tensor m_{ij} is defined as

$$m_{ij} = v_i \otimes v_j (i,\ j = 1,\ 2,\ 3) \tag{1}$$

where $\otimes$ denotes the tensor product or Dyadic product and v_i are the basic vectors that stand for the normal direction of the transverse isotropic plane, namely the sedimentation plane. When the sedimentation plane is parallel to one of the three coordinate surfaces, e.g., x_2–x_3 plane as shown in Fig. 1(a), the normal vector of the sedimentation plane coincides with the x_1 axis. The components of m_{ij} can then be specified as

$$\mathbf{M}_0 = m_{ij}^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

For simplicity, the coordinate axes (x_1,x_2,x_3) shown in Fig. 1 are assumed to be coincident with the direction of the principle stresses.

The sedimentation plane may not be parallel to one of the three coordinate surfaces, e.g., there is an inclined angle θ between the normal vector of the sedimentation plane and x_1 axis, as shown in Fig. 1(b). In this case, the orientation tensor m_{ij} of the sedimentation plane can be obtained through the operation of tensor coordinate transformation from the (x_1,x_2,x_3) coordinate system to the principle stress coordinate system of (x'_1,x'_2,x'_3). The coordinate transformation of orientation tensor can then be expressed as

$$\begin{aligned} \mathbf{M} &= \mathbf{Q}\mathbf{M}_0\mathbf{Q}^{\mathbf{T}} \\ &= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 0 & 0 \\ \sin\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta & 0 & \sin\theta\cos\theta \\ 0 & 0 & 0 \\ \sin\theta\cos\theta & 0 & \sin^2\theta \end{bmatrix} \end{aligned} \tag{3}$$

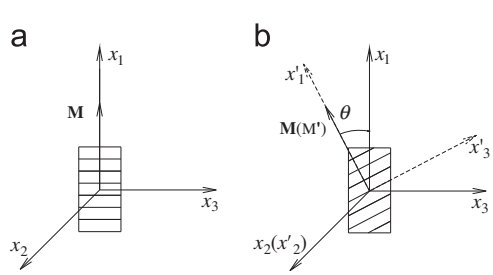

Fig. 1. Orientation tensor of transversely isotropy.

where

$$\mathbf{Q} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

is a transformation tensor when the coordinate system (x_1,x_2,x_3) rotates around x'_2 axis. θ takes a positive value when it is anti-clockwise from the normal vector x'_1 to the principle stress direction x_1.

Following the work by Boehler and Sawczuk (1977), a simplified form of the transformed stress tensor $\hat{\boldsymbol{\sigma}}(\hat{\sigma}_{ij})$ is formulated as follow:

$$\hat{\boldsymbol{\sigma}} = [(\alpha+\gamma-2\beta)\mathrm{tr}(\mathbf{M}\boldsymbol{\sigma})]\mathbf{M} + \gamma\boldsymbol{\sigma} + (\beta-\gamma)(\mathbf{M}\boldsymbol{\sigma}+\boldsymbol{\sigma}\mathbf{M}) \tag{4}$$

where α, β, γ are three independent parameters for the anisotropy. When $\alpha=\beta=\gamma=1$, it means that the material is isotropic. By substituting the transformed stress tensor $\hat{\boldsymbol{\sigma}}(\hat{\sigma}_{ij})$ for the normal stress tensor in isotropic models, it is possible to describe the anisotropic plastic flow of transversely isotropic material.

In triaxial compression tests, the value of the intermediate principal stress and the third principal stress is equal for isotropic materials. For anisotropic materials, however, the stress tensor σ_{ij} in yield surface and the flow rule are substituted by the transformed stress tensor $\hat{\sigma}_{ij}$. Because the intermediate and third principal stress in the transformed stress space is no longer identical, it is much better to use a model capable of properly taking the intermediate-stress dependency in the isotropic material property condition into consideration as the basis of the proposed anisotropic model. Therefore, the t_{ij} concept (Nakai and Mihara, 1984) is adopted in the proposed model.

In present model, the stress tensor has to be transformed twice, the first step is to transform the real stress tensor σ_{ij} to the transformed stress tensor $\hat{\sigma}_{ij}$, namely to form the equivalent isotropic stress that can be expressed by Eq. (4). The second step is to transform the equivalent isotropic stress tensor $\hat{\sigma}_{ij}$ to the stress tensor $\hat{t}_{ij}$ in $\hat{t}_{ij}$ stress space.

2.2. Yield surface and associated flow rule

A graphic description of the transformation approach form σ_{ij} to t_{ij} is shown in Fig. 2 and a brief derivation of the transformed stress tensor t_{ij} is given in Appendix A. A more detailed description of the t_{ij} concept can be found in the works by Nakai and Hinokio (2004).

By the aforementioned twice stress transformation, the stress tensor σ_{ij} can be substituted by the transformed stress tensor $\hat{t}_{ij}$ in the yield surface for anisotropic materials. The yield surface of the proposed anisotropic model in $\hat{t}_{ij}$ stress space is shown in Fig. 3, in which $\hat{t}_N$ and

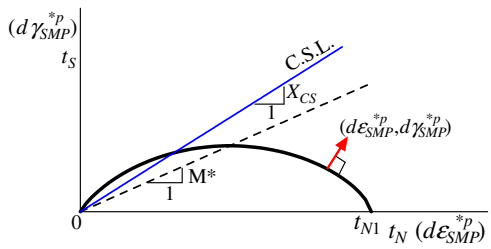

Fig. 2. Yield surface on t_N–t_S plane (after Nakai and Hinokio, 2004).

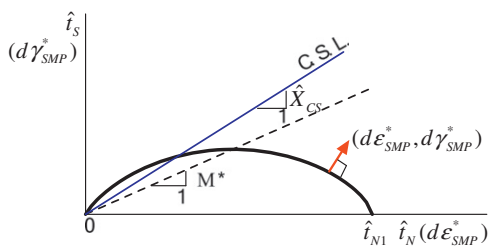

Fig. 3. Yield surface on $\hat{t}_N$–$\hat{t}_S$plane.

$\hat{t}_S$ are the mean stress and the shear stress defined in $\hat{t}_{ij}$ stress space.

In the proposed model, just as was the case in the original model, an associated flow rule is adopted and the plastic potential takes the same form as the original model. Therefore, the yield surface can be expressed in the transformed stress space for normally consolidated geomaterials as

$$f = \ln\frac{\hat{t}_N}{\hat{t}_{N1}} + \zeta(\hat{X}) = \ln\frac{\hat{t}_N}{\hat{t}_{N0}} + \zeta(\hat{X}) - \ln\frac{\hat{t}_{N1}}{\hat{t}_{N0}} = 0 \tag{5}$$

$$\zeta(\hat{X}) = \frac{1}{\beta}\left(\frac{\hat{X}}{M^*}\right)^{\beta} \tag{6}$$

where $\hat{t}_N$ and $\hat{X} = \hat{t}_S/\hat{t}_N$ are the mean stress and the shear stress ratio, respectively, based on t_{ij}-concept, and $\hat{t}_{N1}$ determines the size of the yield surface under the condition of $\hat{t}_S = 0$ in the transformed stress space, as shown in Fig. 3. In order to define a reference state through which the normally consolidated line (NCL) passes, a reference mean stress in the transformed stress space, $\hat{t}_{N0}$, should be given. Without losing generality, $\hat{t}_{N0}$ is an arbitrary value and can be taken as 98 kPa. β is a parameter which determines the shape of the yield surface in transformed stress space.

2.3. Modification of physical quantity at critical state

M^* is a physical quantity related to the critical state. Similar to the original model, as shown in Fig. 4, M^* in Eq. (6) can be expressed using the shear stress ratio $\hat{X}_{CS} = (\hat{t}_S/\hat{t}_N)_{CS}$ and an incremental plastic strain ratio

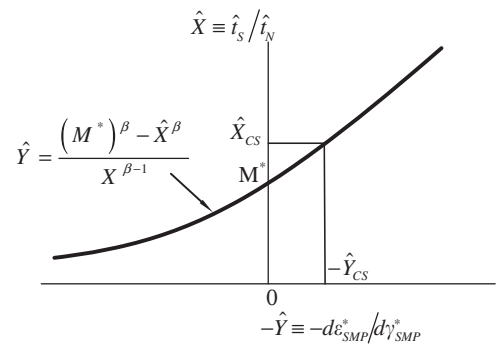

Fig. 4. Relationship between stress ratio and plastic strain increment ratio on $\hat{t}_N$–$\hat{t}_S$ plane.

$\hat{Y}_{CS} \equiv (d\varepsilon_{SMP}{}^{*p}/d\gamma_{SMP}{}^{*p})_{CS}$ at the critical state as follows:

$$M^* = (\hat{X}^{\beta}_{CS} + \hat{X}_{CS}{}^{\beta-1}\hat{Y}_{CS})^{1/\beta} \tag{7}$$

where $d\varepsilon_{SMP}{}^{*p}$ and $d\gamma_{SMP}{}^{*p}$ are the incremental volumetric strain and shear strain in the t_{ij} stress space. The ratios $\hat{X}_{CS}$ and $\hat{Y}_{CS}$, which are equal to the ratios X_{CS} and Y_{CS}, respectively, under isotropic condition, can be evaluated with the following equations as (Nakai and Matsuoka, 1986)

$$\hat{X}_{CS} = \frac{\sqrt{2}}{3}\left(\sqrt{R_{CS}} - \frac{1}{\sqrt{R_{CS}}}\right),$$
$$\hat{Y}_{CS} = \frac{1-\sqrt{R_{CS}}}{\sqrt{2}(\sqrt{R_{CS}}+0.5)} \tag{8}$$

where

$$R_{CS} = (\sigma_1/\sigma_3)|_{at\ \ critical\ state} \tag{9}$$

is the ratio of the principal stresses σ_1 to σ_3 at critical state in conventional triaxial compression test under isotropic condition.

For soft sedimentary rock, it is known that the stress ratio at critical state R_{CS} takes different values at different confining stresses (Adachi and Ogawa,1980; Iwata, 2011). This confining-stress dependency has already been solved properly in p–q stress space in a constitutive model for soft rock proposed by Zhang (1994). In the t_{ij} stress space, however, it is still a problem that has yet to be solved. It is evident that the expression for M^* needs to be modified to consider the influence of confining stress on the physical quantity at critical state as above mentioned. Based on the shear-dilation relationship in t_{ij} stress space derived from conventional triaxial compression tests, the variable M^* is formulated in the following way as

$$M^*_0 = M^*_{CS}\hat{OCR}^n, \quad M^* = M^*_0 + \int dM^* \tag{10}$$

$$dM^* = A\ln\frac{M^*_{CS}}{M^*}d\varepsilon^p_d \tag{11}$$

where M^*_0 is the initial value of M^* at the beginning of shear loading, $\hat{OCR}$ is the overconsolidation ratio of soft rock at initial condition defined in the transformed stress space, and M^*_{CS} is the value of M^*_0 when $\hat{OCR}=1.0$. $d\varepsilon^p_d$ is a

differential of equivalent plastic shear strain during shear loading and will be described in detail later. Two new parameters n and A are introduced: n, the exponential in the relationship between $O\hat{C}R$ and M_0^*, and A, which determines the evolution rate of M^* during the shear loading process. Since M^* is included in the right- and left-hand side of Eq. (11), in the calculation of dM^*, for each very small increment $d\varepsilon_d^p$, the value of M^* is assumed to be constant for ease of calculations without losing satisfactory accuracy.

Using the concept of subloading yield surface (Hashiguchi and Ueno, 1977), the yield surface given by Eq. (5) can be rewritten as

$$f = \ln\frac{t_N}{\hat{t}_{N0}} + \zeta(\hat{X}) - \ln\frac{t_{N1}}{\hat{t}_{N0}} = \ln\frac{t_N}{\hat{t}_{N0}} + \zeta(\hat{X}) - \left(\ln\frac{\hat{t}_{N1e}}{\hat{t}_{N0}} - \ln\frac{\hat{t}_{N1e}}{\hat{t}_{N1}}\right) = 0 \tag{12}$$

where, as is the same as in Fig. 5(a), $\hat{t}_N$ is the present stress state and $\hat{t}_{N1}$ is the cross point of the $\hat{t}_N$ axis with the subloading yield surface that passes through the present stress state. $\hat{t}_{N1e}$ is the value of mean stress $\hat{t}_N$ at the cross point of $\hat{t}_N$ axis of with the normal yield surface. $\hat{t}_{N1e}$ can be uniquely determined by the present plastic volumetric strain in such a way that

$$\varepsilon_v^p = C_p \ln\frac{\hat{t}_{N1e}}{\hat{t}_{N0}}, \quad C_p = \frac{\lambda-\kappa}{1+e_0} \tag{13}$$

where e_0 is the void ratio at the reference state $\hat{t}_N = \hat{t}_{N0} = 98$ kPa under isotropic normal consolidated condition. λ is the compression index and κ is the swelling index. Similar to the original model, the state variable ρ is introduced as shown in Fig. 5(b), which shows a clear relation between ρ and the ratio $\hat{t}_{N1e}/\hat{t}_{N1}$ as follows:

$$\rho = E_p \cdot \ln\left(\frac{\hat{t}_{N1e}}{\hat{t}_{N1}}\right) = (1+e_0)\cdot C_p \cdot \ln\left(\frac{\hat{t}_{N1e}}{\hat{t}_{N1}}\right) = (1+e_0)\cdot C_p \cdot \ln O\hat{C}R \tag{14}$$

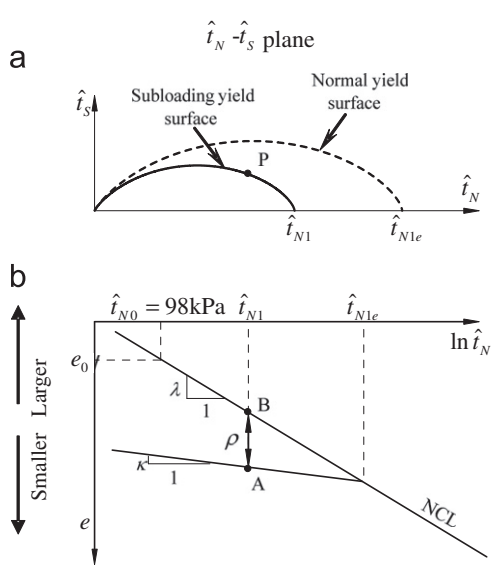

Fig. 5. Shape of yield surfaces and definition of ρ on $\hat{t}_N - \hat{t}_S$ plane.

ρ can be used to represent the relation between the size of the subloading yield surface and the normal yield surface, as shown in Fig. 5. By substituting Eqs. (14) and (13) into Eq. (12), the following yield function can be obtained:

$$f = \ln\left(\frac{\hat{t}_N}{\hat{t}_{N0}}\right) + \xi(\hat{X}) - \frac{1}{C_p}\left(\varepsilon_v^p - \frac{\rho}{1+e_0}\right) = 0 \tag{15}$$

which is the same in the form as the original model (Zhang et al., 2005), but with different variants: $\hat{t}_N$ and $\hat{X}$ are substituted for t_N and X. In the transformed stress space, since the associated flow rule is adopted in the present model, the incremental plastic strain can expressed as

$$d\varepsilon_{ij}^p = \Lambda\frac{\partial f}{\partial t_{ij}}, \quad d\varepsilon_v^p = \Lambda\frac{\partial f}{\partial \hat{t}_{kk}} \tag{16}$$

where Λ is a positive quantity called the plastic factor, and can be subsequently induced by the consistency equation. The consistency equation in the transformed stress space is taken as the same form as that in the isotropic constitutive model and is expressed as

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial M^*} dM^* - \frac{1}{C_p}\left[d\varepsilon_v^p - d\left(\frac{\rho}{1+e_0}\right)\right] = 0 \tag{17}$$

In the above equation, the only difference between the original model and the proposed model is that the term $(\partial f/\partial M^*)dM^*$ is absent in the original model because M^* is a constant.

Similar to the original model, the evolution equation for the state variable ρ is given in the form as

$$\frac{1}{1+e_0} d\rho = -\Lambda\frac{G(\rho)}{\hat{t}_N} \tag{18}$$

where

$$G(\rho) = a\rho^b \tag{19}$$

The parameters a and b determined the evolution rate of ρ. From Eq. (16), the incremental equivalent plastic shear strain $d\varepsilon_d^p$ is expressed as

$$d\varepsilon_d^p = \sqrt{d\varepsilon_{ij}^p d\varepsilon_{ij}^p} = \Lambda\sqrt{\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)} \tag{20}$$

where the Kronecker delta δ_{ij} is the unit tensor.

The plastic factor Λ can be determined with the consistency equation. By substituting Eqs. (10), (18), and (20) into Eq. (17), it is easy to obtain the expression for Λ as

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}}{\frac{h_p}{C_p} - A\ln\frac{M_{CS}^*}{M^*}\frac{\partial f}{\partial M^*}}\sqrt{\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)} \tag{21}$$

where

$$h_p = \frac{1}{C_p}\left(\frac{\partial f}{\partial \hat{t}_{kk}} + \frac{G(\rho)}{\hat{t}_N}\right) \tag{22}$$

As for the elastic strain component, a common way used in normal elasto-plasticity theory is adopted, that is, the

differential of the total strain tensor can be divided into the elastic and the plastic components as

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \tag{23}$$

The elastic response is formulated by the incremental forms of Hooke's theory as

$$d\varepsilon_{kl}^{e} = E_{ijkl}^{-1} d\sigma_{ij}, \quad E_{ijkl} = \Gamma\delta_{ij}\delta_{kl} + G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}),$$

$$\Gamma = vE/(1+v)(1-2v), \quad G = E/2/(1+v) \tag{24}$$

where E is Young's modulus and v is Poisson's ratio. As Young's modulus of soft sedimentary rock usually increases with the increasing of confining stress, a hypoelastic relation is adopted for E as

$$E = \frac{3(1-2v)(1+e_0)}{\kappa}\sigma_m \tag{25}$$

where σ_m is the mean principle stress, κ stands for the swelling index of the soft rock.

Combining Eqs. (17)–(19), (23) and (24) yields a specific consistency equation as

$$df = \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl}$$

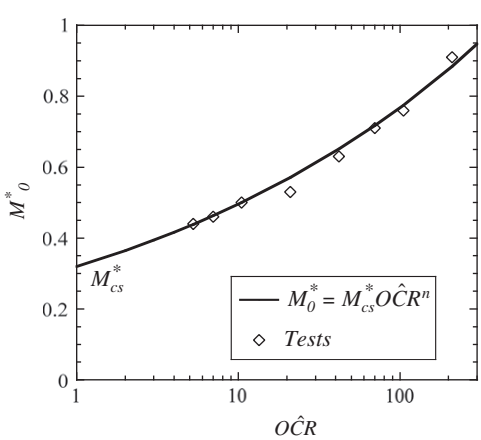

Fig. 6. Relationship between M_0^* and initial overconsolidation ratio $\hat{OCR}$ for Ohya stone.

$$-\Lambda\left[\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \frac{\partial f}{\partial \hat{t}_{kl}} + \frac{1}{C_p}\left(\frac{\partial f}{\partial \hat{t}_{ii}} + \frac{a\rho^b}{\hat{t}_N}\right)\right] = 0 \tag{26}$$

The expression for Λ can then be rewritten as

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl}}{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \frac{\partial f}{\partial \hat{t}_{kl}} + \frac{h_p}{C_p} - A\ln\frac{M^*_{CS}}{M^*}\frac{\partial f}{\partial M^*}\sqrt{\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)\left(\frac{\partial f}{\partial \hat{t}_{ij}} - \frac{\partial f}{\partial \hat{t}_{kk}}\frac{\delta_{ij}}{3}\right)}} \tag{27}$$

where $\partial f/\partial \hat{t}_{ij}$, the differential at the transformed stress space $\hat{t}_{ij}$, can be obtained using the same way as those in t_{ij} space only by substituting the $\hat{t}_{ij}$ tensor for t_{ij} tensor in the expression of $\partial f/\partial t_{ij}$, which was described in detail in the work by Nakai and Hinokio (2004). While the deductions of $\partial f/\partial \sigma_{ij}$ is described in detail in Appendix B.

The loading criteria are given in the same way as the work by Zhang et al. (2007) shown below

$$\begin{cases} \Lambda > 0 & \textit{plastic loading} \\ \Lambda = 0 & \textit{neutral} \\ \Lambda < 0 & \textit{elastic unloading} \end{cases} \tag{28}$$

In the case of plastic loading, the denominator is positive, that is $\Lambda > 0$. There exist two different situations listed as follow:

$$\begin{cases} \Lambda = \dfrac{numerator > 0}{denomin\ ator > 0} > 0 & \text{strain}-\text{hardening} \quad (i) \\ \Lambda = \dfrac{numerator < 0}{denomin\ ator < 0} > 0 & \text{strain}-\text{softening} \quad (ii) \end{cases} \tag{29}$$

From Eq. (29), it is very easy to define the strain-hardening and the strain softening, that is, situation (i) represents the strain hardening because the yield surface is expending while situation (ii) represents the strain softening because the yield surface is shrinking.

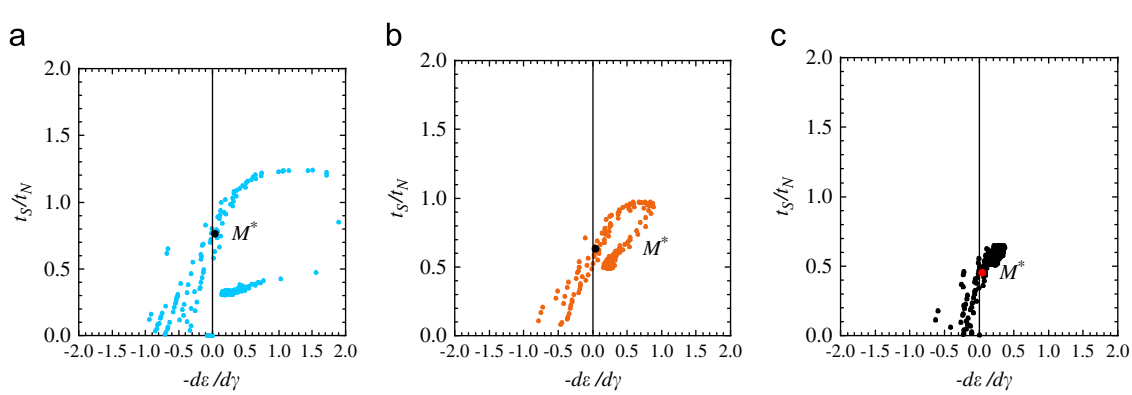

Fig. 7. Test results of stress–dilatancy relation obtained from drained triaxial compression test under different confining pressures. (a) σ_3=0.1 Mpa, (b) σ_3=1.0 Mpa and (c) σ_3=4.0 Mpa1

3. Performance of the model on soft sedimentary rocks

3.1. Determination of material parameters involved in the proposed model

Compared with the original model, two additional sets of parameters are introduced in the anisotropic model. The first set of parameters α, β and γ, determines the magnitude of the anisotropy due to the stratification of soft sedimentary rock formed in its sedimentary process. When $\alpha=\beta=\gamma=1.0$, the model just degenerates to an isotropic model. The values of α, β and γ can be determined from curve fitting method, in which the variation tendency of the peak strengths of the test specimens in drained triaxial test is used to find out a most fitted values with different angle θ.

Photo 1. Test device for Ohya stone under drained triaxial compression test.

The second set of parameters A, n and M^*_{CS}, replacing the original parameter of principal stress ratio at critical state R_{cs} in the original model, is used to evaluate the physical quantity M^* at the critical state.

Fig. 6 shows the comparison of the test and the simulated relationship between M^*_0 and overconsolidation ratio $O\hat{C}R$, in which the simulated one is evaluated by Eq. (10). Detailed information about the test will be given in next section. As can be seen in the figure, M^*_0 increases as the overconsolidation ratio $O\hat{C}R$ increases at the initial state. Eq. (10) agrees well with the test results, which indicates that it is easy to determine the values of the parameters M^*_{CS} and n.

The parameter A is also determined based on the test results of drained triaxial compression test under different confining stresses. Fig. 7 shows the test results of stress–dilatancy relation of Ohya stone obtained from the drained triaxial compression test under different confining stresses. In the figure, M^* decreases with increasing confining stress at the critical state. On the other hand, the value of M^* is calculated with Eqs. (7)–(11). Based on the test results in this figure, it is very easy to obtain the value of R_{CS} under

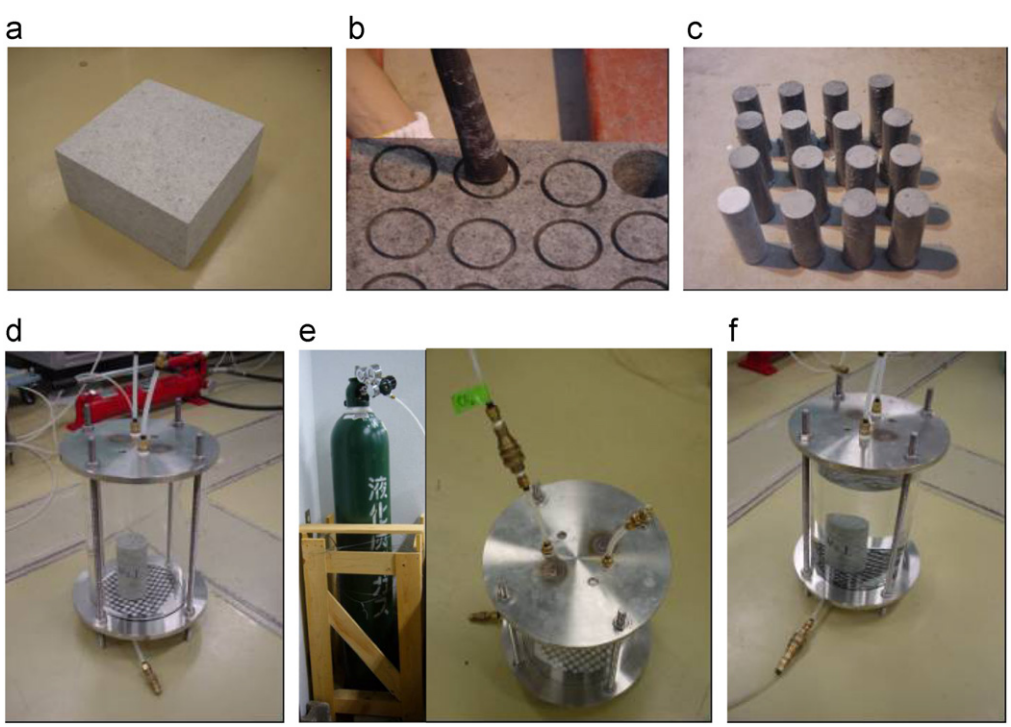

Photo 2. Preparation of the saturated test specimens (B value >0.95). (a) Block sampling of Ohya stone. (b) Coring of samples. (c) Samples before polish. (d) Evacuation in desiccator for certain time. (e) Injection with carbon dioxide gas. (f) Replaced with gas-free water and evacuated again for certain time.

different confining stresses and consequently the value of parameter A.

Parameter b, which controls the evolution of the over-consolidation ratio $\hat{OCR}$ during shearing, should be determined by curve fitting between the test and simulated peak strength in stress–strain relationship. In the original model, M^* is assumed to be constant and therefore the peak strength is only controlled by the parameter b. In the present model, however, the peak strength is also affected by the value of M^*, which is no longer a constant value. Therefore, before determining the value of the parameters b, the parameters A, M^*_{CS} and n should be determined first in the manner already presented in this paper.

3.2. Drained triaxial compression test for Ohya stone under different confining-stresses but the same sedimentary direction

The soft sedimentary rock specimens used in this study, Ohya stone, is a porous tuff deposited in the Miocene Epoch of the Tertiary Period, mined by block sampling at a depth of 30 m in Tochigi Prefecture, Japan. Many experimental studies on Ohya Stone can be found in the literature, such as the works by Adachi and Ogawa (1980), Adachi and Takase (1981), Koike (1997) and Kobayashi (2000).

Drained triaxial compression tests on soft sedimentary rock, Ohya stone and Tomuro stone, which is also a kind of Ohya stone but with much less impurity, are simulated to examine the performance of the proposed model for the mechanical behavior of the soft sedimentary rock. Photo 1 shows the triaxial compression test apparatus with a fully computer-controlled loading device. The capacity of the apparatus is 5 MPa in confining stress and 50 kN in vertical loading applied by a air actuator. The specimen is first isotropically consolidated to a prescribed confining stress and then a strain-rate-controlled vertical load is applied to the specimen under drained condition in conventional compression test. During shearing, the loading rate is 0.001% min, which is sufficient to assure an apparent drained condition (Ye et al., 2007). In the test, the axial strain was measure with inner foucault current transducer with an accuracy of 1 μ.

The specimen was 100 mm in height and 50 mm in diameter, as shown in Photo 2. Before testing, the specimen was saturated through repeated process of (d) evacuation in desiccator; (e) injection with carbon dioxide gas (CO_2); and (f) replacing with gas-free water, as shown in Photo 2. By above saturation process, the value of coefficient B of the samples can be kept to a value larger than 0.95, which is the ratio of an incremental cell pressure to the corresponding excessive pore water pressure measured at the ends of the sample under isotropic conditions. Eight sets of specimens under different confining stresses from 0.1 MPa to 4.0 MPa were used in the drained triaxial compression tests.

In the tests, in order to check the capability of the model to describe the confining-stress influence on the mechanical behavior at critical state with a fixed set of material

Table 1
Physical properties of Ohya stone and Tomuro stone.

Parameters	Ohya stone	Tomuro stone
Initial void ratio e_i	0.57	0.62
Specific gravity of soil particles G_s	2.51	2.53
Consolidation yielding stress p'_c (MPa)	21.0	8.8

Table 2
Material parameters for Ohya stone.

e_0	λ	κ	a	b	β	ν	M^*_{cs}	n	A
0.570	0.018	0.005	850.0	1.10	1.10	0.020	0.32	0.19	4.0

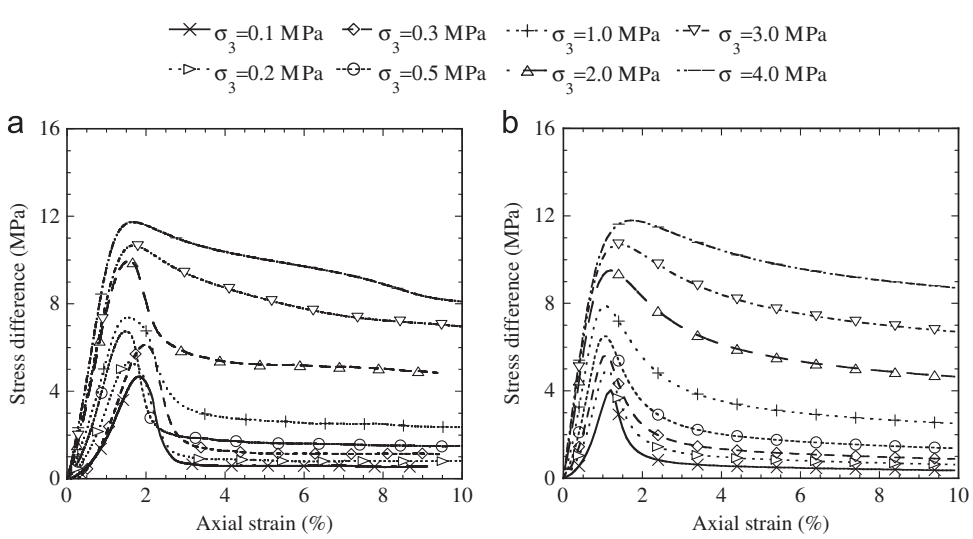

Fig. 8. Verification of the predicted stress–strain relation for Ohya stone under different confining pressures drained triaxial compression tests. (a) Tests and (b) theory.

parameters, the eight sets of specimens were all prepared in the same manner, that is, the sedimentation strata was placed in the horizontal direction and $\theta = 0^{\circ}$. Therefore, the anisotropy due to the inclination angle of strata θ was not considered in simulating these tests, and correspondingly $\alpha = \beta = \gamma = 1.0$ is adopted in the simulation of the tests. The physical properties and the material parameters involved in the simulation are listed in Tables 1 and 2. It is known from Table 2 that the values of the material parameters are all fixed irrespective of the fact that the consolidation ratios at the beginning of shear loading $O\hat{C}R$ are all different with different confining stresses.

The comparison of stress–strain relations and stress–dilatancy relations between the simulated and test results is shown separately in Figs. 8 and 9. It can be seen from the figures that the simulated response is in good agreement with the experimental data under different confining stresses from 0.1 MPa to 4.0 MPa. Once again it needs to be emphasized that all the values of the material parameters are identical under different confining stresses. As for the consolidation ratio at the beginning of shear loading $O\hat{C}R$, it can be easily calculated with the ratio of the confining stress to the pre-consolidation pressure p_c' listed in Table 1.

As to the determination of p_c', not like a clay in which the pre-consolidation pressure is simply the maximum stress that the soil experienced, the value p_c' here is just a reference stress by which it is possible to uniquely define the $O\hat{C}R$ of soft rock based on the assumption that the soft rock can be regarded as a heavily over-consolidated clay. Because the value p_c' is a constant, the rationality of the assumption is assured.

3.3. Drained triaxial compression test for Tomuro stone under different confining-stresses and different sedimentary directions

In succession, test results obtained from the work by Kobayashi (2000) are simulated to examine the capability of the proposed model to describe properly the anisotropy of soft sedimentary rock. The specimens using in these tests is Tomuro stone. There are three series of drained triaxial tests under confining stresses of 0.19 MPa, 0.59 MPa and 0.98 MPa separately, to be simulated and demonstrated in this section. The specimens were cut and

Table 3
Material parameters for Tomuro stone.

e_0	e_0	λ	a	b	β	v	M^*_{cs}	n	A
0.618	0.019	0.003	850.0	1.10	1.05	0.02	0.38	0. 20	1.0

Table 4
The anisotropic parameters adopted in the model for Tomuro stone.

α	0.95
β	1.05
γ	0.98

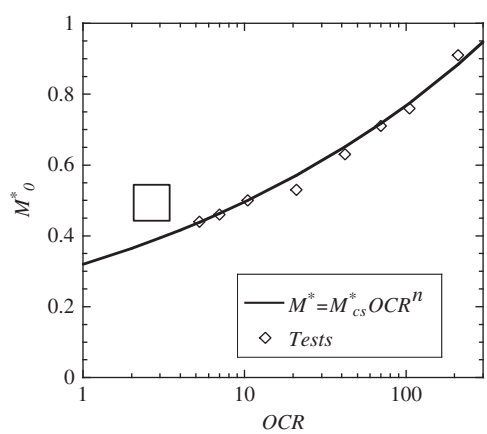

Fig. 10. Relationship between $M^*$and overconsolidation ratio $O\hat{C}R$ for Tomuro stone.

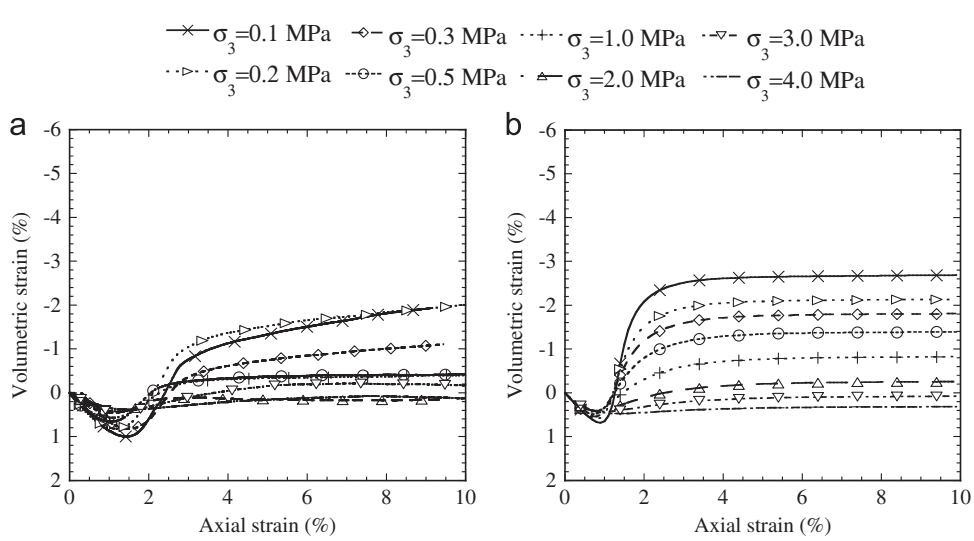

Fig. 9. Verification of the predicted stress–dilatancy relation for Ohya stone under different confining pressures drained triaxial compression tests. (a) Tests and (b) theory.

placed on the attitudes with five different inclination angles θ under each confining stress. Detailed description about the tests can be referred to the works by Kobayashi (2000) and Oka et al. (2002). Tables 3 and 4 list the material parameters and the anisotropic parameters involved in the model. Because the overconsolidation ratio $O\hat{C}R$ is defined in the transformed stress space and α, β and γ are no longer equal to 1.0, the relation between M^* and $O\hat{C}R$ under different confining stresses in real stress space can only be easily calculated in the case of $\theta = 0^\circ$. Therefore, the value of M_{cs} is calculated from the relation M^* and $O\hat{C}R$ under the condition of $\theta = 0^\circ$. In spite of the way to determine the value of M_{cs}, the material parameters are all fixed under different angles θ and different confining stresses. Fig. 10 shows the comparison of the test and the simulated relationship between M^* and the overconsolidation ratio $O\hat{C}R$ under the condition of $\theta = 0^\circ$, in which the simulated one is also evaluated by Eq. (10), just as was the case in Fig. 7.

Figs. 11–13 show the comparison between the predicted and the test stress–strain relations and stress–dilatancy relations under three different confining stresses. It is found that the stress–strain relation is on the whole reproduced by the model in all the tests, with different confining stresses and different sedimentary angles, which

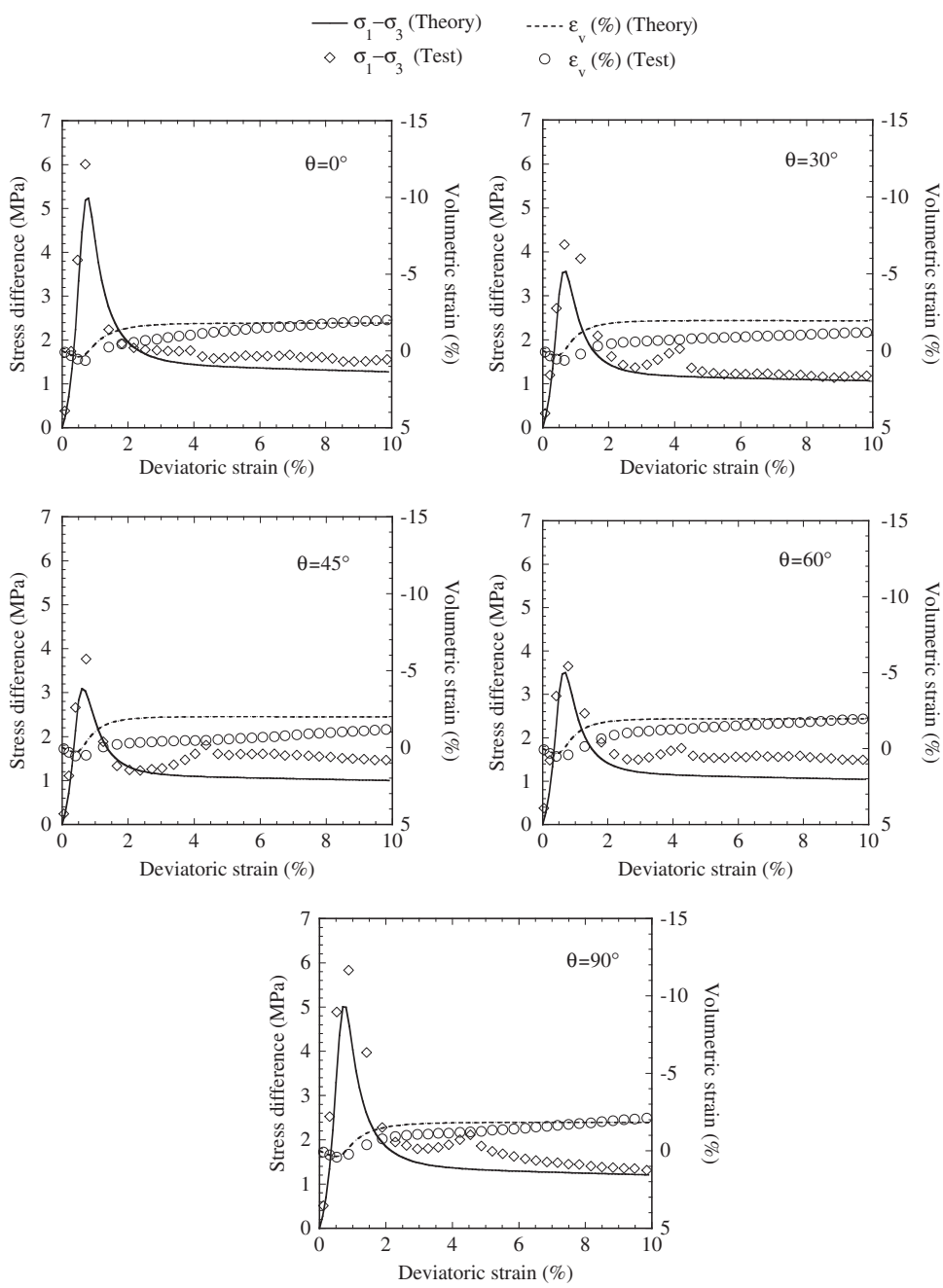

Fig. 11. Verification of the predicted stress–strain relation and stress–dilatancy relation for Tomuro stone under 0.19 MPa confining pressure drained triaxial compression tests (test data by Kobayashi, 2000).

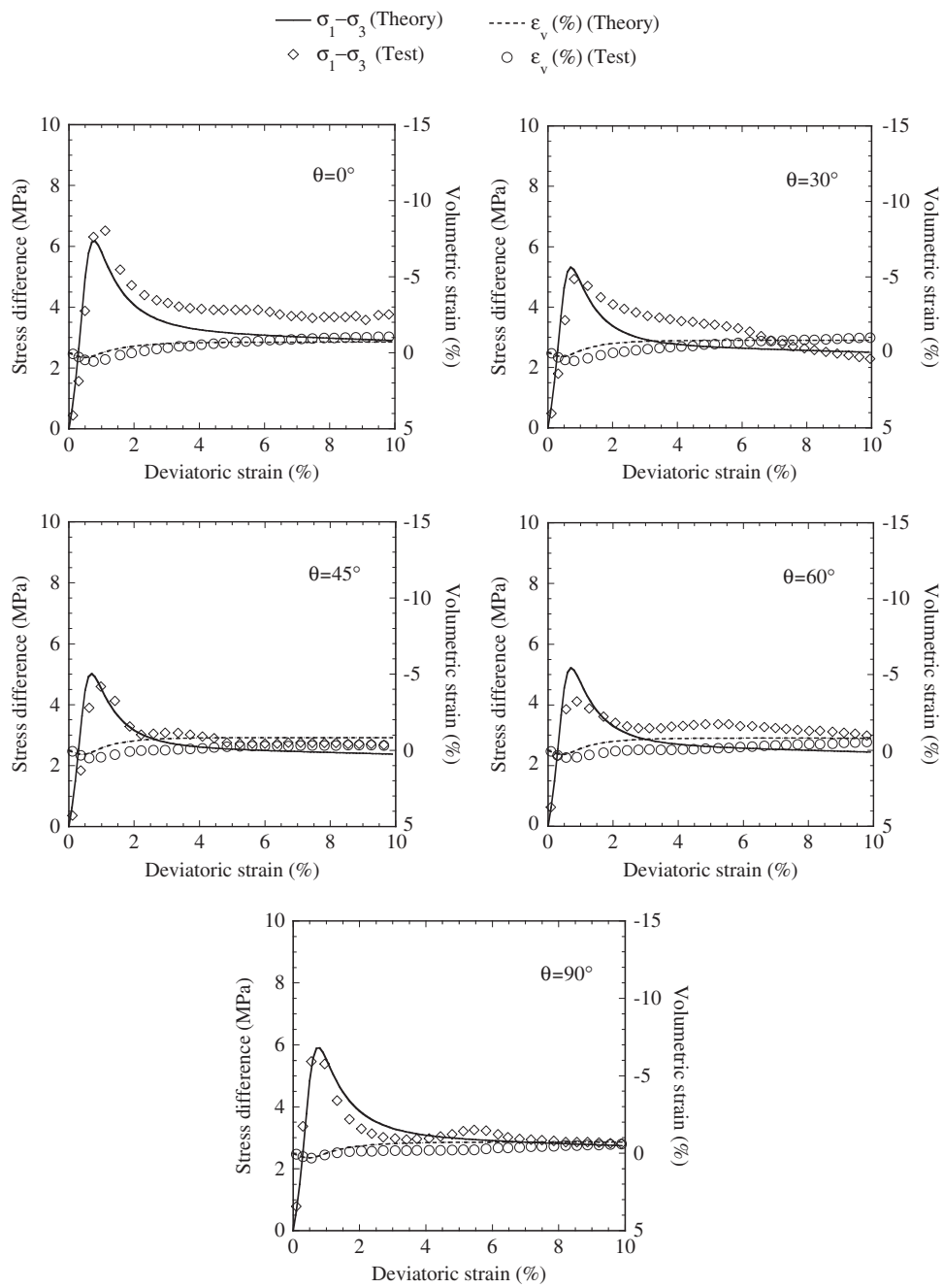

Fig. 12. Verification of the predicted stress–strain relation and stress–dilatancy relation for Tomuro stone under 0.59 MPa confining pressure drained triaxial compression tests (test data by Kobayashi, 2000).

has a more general meaning in describing the mechanical behavior of the soft rock, if compared with the works by Oka et al. (2002), where only one set of the test results under one confining stress was discussed.

The comparisons of the variation tendency in peak strength with respect to θ under different confining stresses are shown in Fig. 14. It can be seen from the figure that the maximum strength appears at $\theta = 0^\circ$ and $\theta = 90^\circ$, both in the tests and in the theoretical simulation. While for the minimum strength, it appears at the angle around $\theta = 45^\circ$, also both in the tests and in the theoretical simulation. The overall behavior of the predicted variation tendency coincides well with the test results.

4. Conclusions

In this paper, an anisotropic constitutive model for soft sedimentary rock is proposed. Compared with other constitutive models in the literature, the new model can describe the general mechanical behavior of soft rock in a more accurate and comprehensive way. The validity and

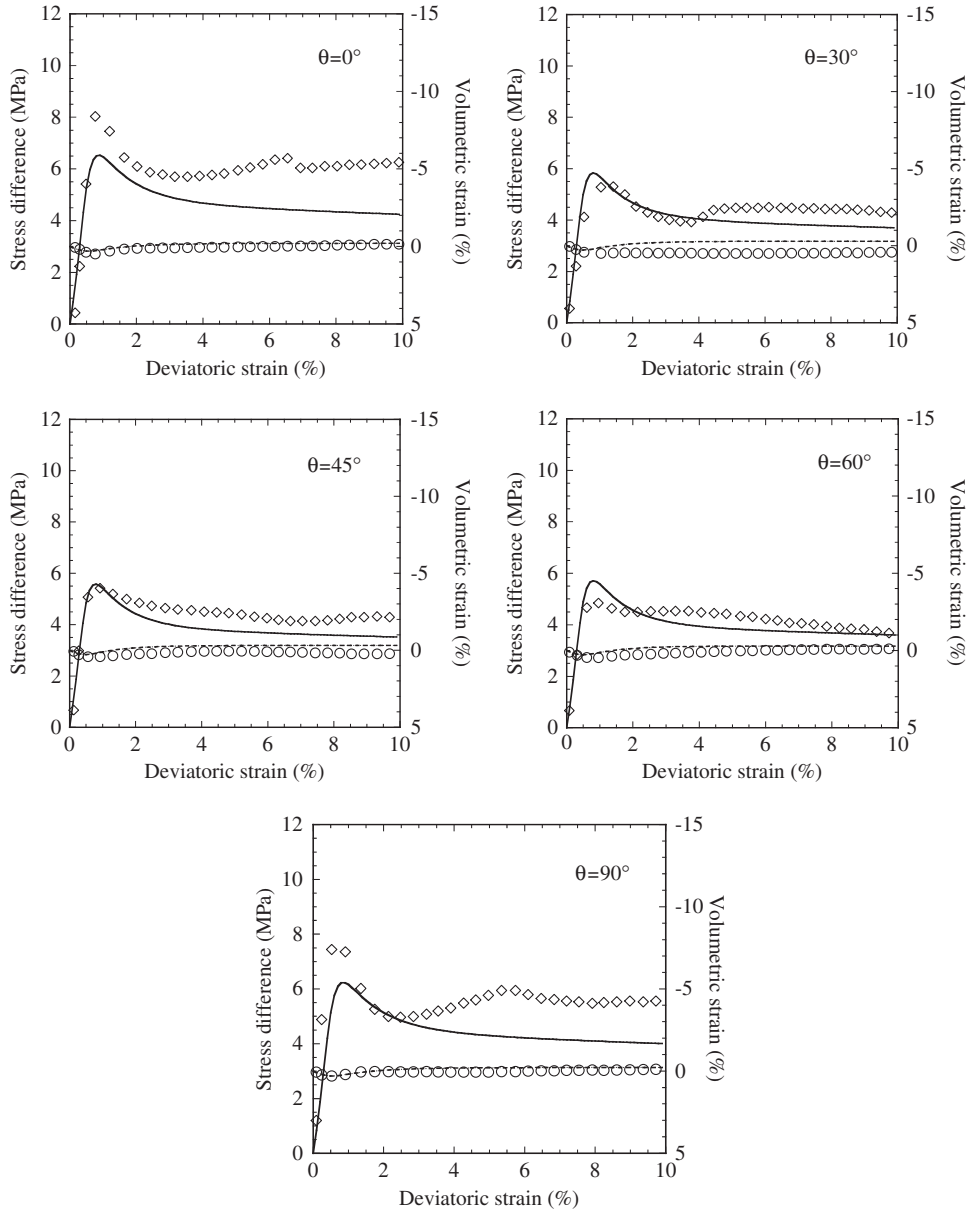

Fig. 13. Verification of the predicted stress–strain relation and stress–dilatancy relation for Tomuro stone under 0.98 MPa confining pressure drained triaxial compression tests (test data by Kobayashi, 2000).

the capability of the model are verified with laboratory test data. The essence of the model is summarized as follows:

1. In the pioneer works by Kobayashi (2000) and Oka et al. (2002), the inherent anisotropic mechanical behavior of soft sedimentary rock due to the sedimentation strata has been investigated both in laboratory tests and constitutive model. Nevertheless, some important problems remain that need to be solved, e.g., the confining-stress dependency and the intermediate-stress dependency. By introducing a new evolution equation for the critical state parameter M^* under different confining stresses, the newly proposed model is possible to describe the confining-stress dependency of the soft sedimentary rock, with a set of new material parameters whose values are identical under all loading condition.
2. Using Boehler's theory, a transformed stress tensor is introduced into the original model (Zhang et al., 2005). The results indicate that the inherent anisotropic mechanical behavior of soft sedimentary rock can be described in a unified way, and that the peak strength, the residual strength, the stress–strain relation and stress–dilatancy relation in different sedimentary directions can be reproduced with satisfactory accuracy. It should be noted that all the material parameters are kept constant for all sedimentary directions and loading conditions.
3. The intermediate-stress dependency can be properly described due to the adoption of the t_{ij} concept. In addition, all the material parameters involved in the model have clear physical meanings and can be

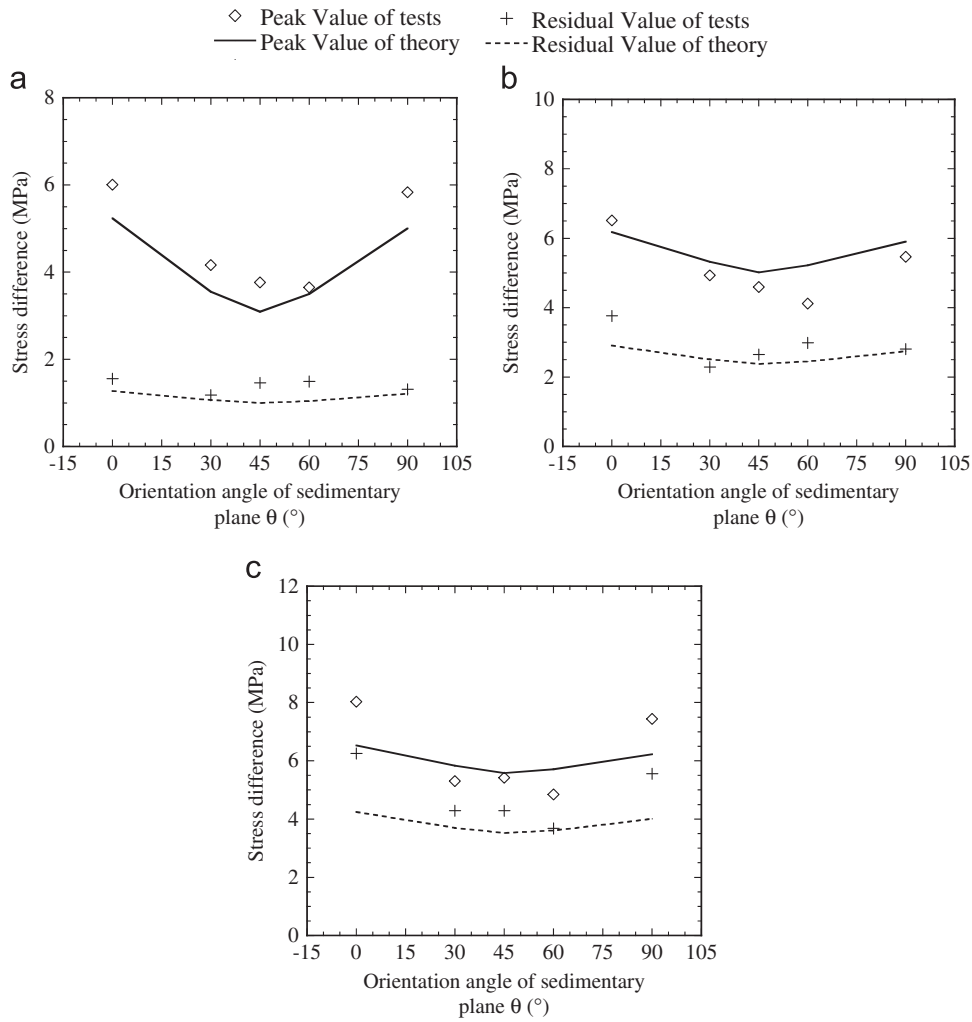

Fig. 14. Variation tendency verification of peak strength with respect to θ for Tomuro stone under drained triaxial compression tests. (a) Confining pressure 0.19MPa, (b) Confining pressure 0.59MPa and (c) confining pressure 0.98MPa.

definitely determined using a consolidation test and a drained conventional triaxial compression test.

4. The deduction of the model is quite simple and has clear physical meaning. All the essential features in the original model, e.g., the subloading yield surface by which it is possible to describe the overconsolidated behavior of geomaterials; the associated flow rule that is much preferred due to its simple and smart form in a constitutive model; and the assumption that only plastic volumetric strain is the hardening parameter in dealing with the strain harden and strain soften, have been smoothly inherited to the newly proposed model.

Appendix A. Definition of t_{ij} (Nakai and Mihara, 1984)

As shown in Fig. A1, the Spatial Mobilized Plane (SMP), defined by Nakai and Matsuoka, can be expressed in a principal stress space (σ_I, σ_{II}, σ_{III}) as

$$\frac{\sigma_I}{\sqrt{\sigma_1}}+\frac{\sigma_{II}}{\sqrt{\sigma_2}}+\frac{\sigma_{III}}{\sqrt{\sigma_3}}=1 \tag{A.1}$$

Therefore, the normal of the plane (a_1, a_2, a_3) can be evaluated by the following equation as

$$a_i=\sqrt{\frac{I_3}{I_2\sigma_i}} \quad (i=1,2,3) \tag{A.2}$$

where I_1, I_2 and I_3 are the first, second and third invariants of effective stress tensor and can be expressed by the following forms using three principal stresses:

$$\begin{aligned} I_1&=\sigma_1+\sigma_2+\sigma_3\\ I_2&=\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\\ I_3&=\sigma_1\sigma_2\sigma_3 \end{aligned} \tag{A.3}$$

In t_{ij} clay and sand models, a symmetric tensor t_{ij} is expressed by a product of stress tensor σ_{ik} and a tensor a_{kj}

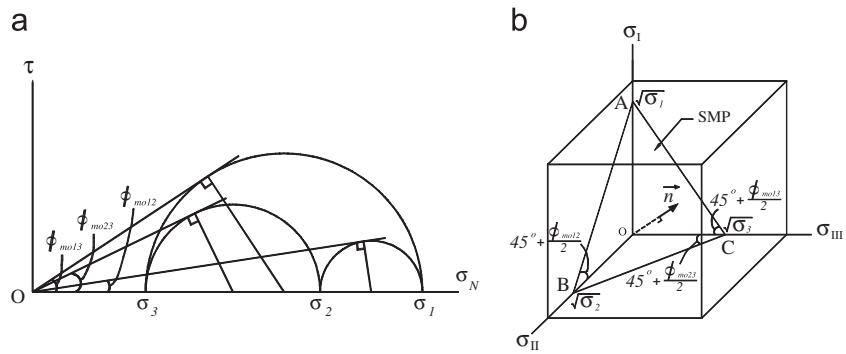

Fig. A1. Explanation of SMP (Nakai and Mihara, 1984). (a) Explanation of mobilized angle ϕ and (b) explanation of SMP.

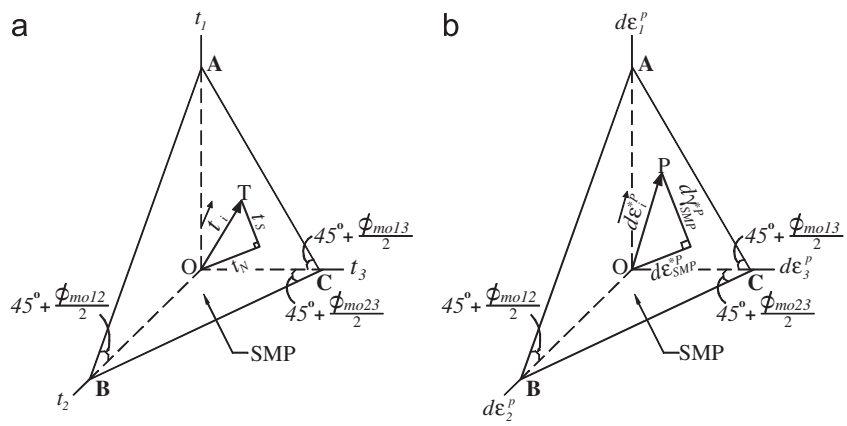

Fig. A2. Explanation of stress and strain in principal-value space of tensor t_{ij} (Nakai and Mihara, 1984). (a) t_s and t_n expressed in principal stress space of t_{ij} tensor and (b) Increment strain expressed in principal increment strain space of $d\varepsilon_{ij}$.

as

$$t_{ij} = \sigma_{ik} a_{kj} \tag{A.4}$$

where tensor a_{kj} can be evaluated by Cayley–Hamilton's theory as

$$a_{ij} = \sqrt{\frac{I_3}{I_2}} r_{ij}^{-1},\ r_{ik} r_{kj} = \sigma_{ij},\ r_{ij} = (\sigma_{ik} + I_2 \delta_{ik})(I_1 \sigma_{kj} + I_3 \delta_{kj})^{-1} \tag{A.5}$$

Because r_{ij} is a one-half power function of σ_{ij}, r_{ij} is a symmetric tensor and its principal directions are the same as σ_{ij} and have the principal values of ($\sqrt{\sigma_1}$, $\sqrt{\sigma_2}$, $\sqrt{\sigma_3}$). Therefore, a_{ij} is also a symmetric tensor and has the same principal directions as σ_{ij} and has the principal value of (a_1, a_2, a_3). As a result, the tensor t_{ij} is also a symmetric tensor and has the same principal directions as a_{ij} and σ_{ij}

A normal component t_n and a tangential component t_s of the principal-value vector of t_{ij} can be given as (see Fig. A2a)

$$t_N = t_1 a_1 + t_2 a_2 + t_3 a_3 = t_{ij} a_{ij} \equiv \sigma_{SMP} = \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2 = 3\frac{I_3}{I_2} \tag{A – I – 6}$$

$$t_S = \sqrt{(t_1^2 + t_2^2 + t_3^2) - t_N^2} = \sqrt{t_{ij} t_{ij} - (t_{ij} a_{ij})^2} \equiv \tau_{SMP} = \sqrt{(\sigma_1 - \sigma_2)^2 a_1^2 a_2^2 + (\sigma_2 - \sigma_3)^2 a_2^2 a_3^2 + (\sigma_3 - \sigma_1)^2 a_3^2 a_1^2} = \frac{\sqrt{I_1 I_2 I_3 - 9 I_3^2}}{I_2} \tag{A.7}$$

It is assumed that the directions of plastic principal strain increments coincide with those of principal axes of t_{ij}, and the plastic strain increment can also be given by a normal component ($d\varepsilon_{SMP}^{*P}$) and a tangential component ($d\gamma_{SMP}^{*p}$) of the principal plastic strain increment vector on the SMP (see Fig. A2b)

$$d\varepsilon_{SMP}^{*P} = d\varepsilon_1^P a_1 + d\varepsilon_2^P a_2 + d\varepsilon_3^P a_3 = d\varepsilon_{ij}^P a_{ij} \tag{A.8}$$

$$d\gamma_{SMP}^{*p} = \sqrt{(d\varepsilon_1^{p2} + d\varepsilon_2^{p2} + d\varepsilon_3^{p2}) - d\varepsilon_{SMP}^{*p2}} = \sqrt{d\varepsilon_{ij}^p d\varepsilon_{ij}^p - (d\varepsilon_{ij}^p a_{ij})^2} \tag{A.9}$$

Appendix B. Deductions of differential $\partial f / \partial \sigma_{ij}$

$\partial f / \partial \sigma_{ij}$ can be written as

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial \hat{\sigma}_{mn}} \frac{\partial \hat{\sigma}_{mn}}{\partial \sigma_{ij}} \tag{B.1}$$

While $\partial f / \partial \hat{\sigma}_{mn}$can be obtained by just substituting the σ_{ij}, stress tensor in normal stress space and t_{ij}, stress tensor in normal t_{ij} stress space with relevant $\hat{\sigma}_{ij}$ tensor and $\hat{t}_{ij}$ tensor

in the expression of $\partial f/\partial\sigma_{mn}$, which was described in detail in the work by Nakai and Hinokio (2004).

In order to calculate $\partial\hat{\sigma}_{mn}/\partial\sigma_{ij}$, the transformation stress tensor $\overset{\sigma}{_{mn}}$ expressed with Eq. (4) can be rewritten in suffix form as

$$\overset{\sigma}{_{mn}} = (\alpha+\gamma-2\beta)m_{kl}\sigma_{lk}m_{mn}+\gamma\sigma_{mn}+(\beta-\gamma)(m_{mk}\sigma_{kn}+\sigma_{mk}m_{kn}) \tag{B.2}$$

Therefore

$$\begin{aligned}\partial\frac{\overset{\sigma}{_{mn}}}{\partial\sigma_{ij}} &= (\alpha+\gamma-2\beta)\frac{\partial(m_{kl}\sigma_{lk}m_{mn})}{\partial\sigma_{ij}}+\gamma\frac{\partial\sigma_{mn}}{\partial\sigma_{ij}} \\ &\quad +(\beta-\gamma)\left[\frac{\partial(m_{mk}\sigma_{kn})}{\partial\sigma_{ij}}+\frac{\partial(\sigma_{mk}m_{kn})}{\partial\sigma_{ij}}\right] \\ &= (\alpha+\gamma-2\beta)\left(\frac{\partial m_{kl}}{\partial\sigma_{ij}}\sigma_{lk}m_{mn}+m_{kl}\delta_{li}\delta_{kj}m_{mn}+m'_{kl}\sigma_{lk}\frac{\partial m_{mn}}{\partial\sigma_{ij}}\right) \\ &\quad +\gamma\delta_{mi}\delta_{nj}+(\beta-\gamma)\left[\frac{\partial m_{mk}}{\partial\sigma_{ij}}\sigma_{kn}+m_{mk}\delta_{ki}\delta_{nj}\right. \\ &\quad \left.+\delta_{mi}\delta_{kj}m_{kn}+\sigma_{mk}\frac{\partial m_{kn}}{\partial\sigma_{ij}}\right]\end{aligned} \tag{B.3}$$

Being the orientation tensor of the sedimentation plane, m_{ij} is a constant with respect to the stress tensorσ_{ij}, it is valid that

$$\frac{\partial m_{kl}}{\partial\sigma_{ij}} = \frac{\partial m_{mn}}{\partial\sigma_{ij}} = \frac{\partial m_{mk}}{\partial\sigma_{ij}} = \frac{\partial m_{kn}}{\partial\sigma_{ij}} = 0 \tag{B.4}$$

Consequently, $\partial\overset{\sigma}{_{mn}}/\partial\sigma_{ij}$ can be expressed as

$$\begin{aligned}\partial\frac{\overset{\sigma}{_{mn}}}{\partial\sigma_{ij}} &= (\alpha+\gamma-2\beta)m_{kl}\delta_{li}\delta_{kj}m_{mn}+\gamma\delta_{mi}\delta_{nj}+(\beta-\gamma) \\ &\quad (m_{mk}\delta_{ki}\delta_{nj}+\delta_{mi}\delta_{kj}m_{kn}) \\ &= (\alpha+\gamma-2\beta)m_{ji}m_{mn}+\gamma\delta_{mi}\delta_{nj}+(\beta-\gamma)(m_{mi}\delta_{nj}+\delta_{mi}m_{jn})\end{aligned} \tag{B.5}$$

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